Code: 15A05302 **R15**

B.Tech II Year I Semester (R15) Regular Examinations November/December 2016

DISCRETE MATHEMATICS

(Common to CSE & IT)

Time: 3 hours Max. Marks: 70

PART - A

(Compulsory Question)

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1 Answer the following: (10 X 02 = 20 Marks)

(a) Construct the truth table $7(7P \vee 7Q)$.

(b) Prove that $(\exists x)(P(x) \wedge Q(x)) \Rightarrow (\exists x)P(x) \wedge (\exists x)Q(x)$.

(c) Let $X = \{1, 2, \ldots \ldots 7\}$ and $R = \{< x, y > | x - y \; is \; divisible \; by \; 3\}$ show that R is an equivalence relation. Draw the graph of R.

(d) Let $\lfloor\sqrt{x}\rfloor$ be the greatest integer $\leq \sqrt{x}$. Show that $\lfloor\sqrt{x}\rfloor$ is primitive recursive.

(e) Let $< G, \star >$ be a finite cyclic group generated by an element $a \epsilon G$. Prove that if G is of order n, i.e $|G| = n$ then $a^n = e$ so that $G = \{a, a^2, a^3, \ldots \ldots a^n = e\}$. Furthermore n is the least +ve integer for which $a^n = e$.

(f) Prove that the minimum weight of the nonzero code words in a group code is equal to its minimum distance.

(g) Let $< L, \leq >$ be a lattice. For any $a, b, c \in L$ then prove that $b \leq c \Rightarrow a * b \leq a * c$.

(h) Prove the Boolean identity $(a \wedge b) \vee (a \wedge b') = a$.

(i) Prove that a tree with n vertices has precisely n-1 edges.

(j) A label identifier for a computer program consists of one letter followed by three digits. If repetitions are allowed, how many distinct label identifiers are possible.

PART - B

(Answer all five units, 5 X 10 = 50 Marks)

UNIT - I

2 (a) Show that the formula $Q \vee (P \wedge 7Q) \vee (7P \wedge 7Q)$ is a tautology.

(b) Obtain the principal conjuctive normal form of the formula given by $(7P \rightarrow R) \wedge (Q \rightleftarrows P)$.

OR

3 (a) Show that $R \wedge (P \vee Q)$ is a valid conclusion from the premises $P \vee Q$, $Q \rightarrow R$, $P \rightarrow M$ and $\neg M$.

(b) Show that $\neg P(a, b)$ follows logically from $(x)(y)\ (P(x, y)) \rightarrow W(x, y) \; and \; \neg \dot{W}(a, b)$.

UNIT - II

4 (a) Let Z be the set of integers and let R be the relation called congruence modulo 3 defined by; $R = \{< x, y > | x \in z \wedge y \in z \wedge (x - y) is \; divisible \; by \; 3\}$. Determine the equivalence classes generated by the elements of z.

(b) Let $X = \{2, 3, 6, 12, 24, 36\}$ and the relation $\leq$ be such that $x \leq y$ if x divides y. Draw the Hasse diagram of $< x, \leq >$.

OR

5 Let F_x be the set of all one to one, onto mappings from X onto X where $X = \{1, 2, 3\}$. Find all the elements of F_x and find the inverse of each element.

UNIT - III

6 (a) Show that every cyclic group of order n is isomorphic to the group $< z_n, t_n >$.

(b) Prove that a subset $S \neq \emptyset$ of G is a subgroup of $< G, * >$. If for any pair of elements $a, b \; \epsilon \; s$, $a * b^{-1} \epsilon s$.

OR

7 Prove that a code can correct all combinations of K or fewer errors if and only if the minimum distance between any two code words is at least 2K+1.

UNIT - IV

8 Prove that a connected graph G is Euler if and only if all the vertices of G are even degree.

OR

9 Explain travelling sales man's problem.

UNIT - V

10 In how many ways can the 26 letters of the alphabet be permitted so that none of the patterns car, dog, pun or byte occurs?

OR

11 Use generating functions to determine how many four element subsets of $S = \{1, 2, 3, \ldots \ldots 15\}$ contain no consecutive integers.

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