

PROBABILITY THEORY & STOCHASTIC PROCESSES

(Electronics and Communication Engineering)

Time: 3 hours

Max. Marks: 70

PART – A

(Compulsory Question)

1 Answer the following: (10 X 02 = 20 Marks)

- (a) State the properties of conditional density function.
 (b) What is the importance of Rayleigh distribution function?
 (c) The joint density function of two discrete random variables X and Y is:

$$f_{XY}(x,y) = kxy; \text{ for } 0 < x < 4, 1 < y < 5$$

$$= 0; \text{ otherwise}$$

Find the value of the constant k.

- (d) Define joint characteristic functions of two random variables.
 (e) Distinguish between stationary and non-stationary random process.
 (f) When two different random processes are said to be statistically independent?
 (g) If the PSD of x(t) is $S_{XX}(\omega)$. Find the PSD of $\frac{dx(t)}{dt}$.
 (h) State any two differences between random variable and random process.
 (i) A wide sense stationary random process x(t) is applied to the input of an LTI system whose impulse response is $5t e^{-2t}$. The mean of x(t) is 3. Find the mean output of the system.
 (j) Give any two spectral characteristics of the system response.

PART – B

(Answer all five units, 5 X 10 = 50 Marks)

UNIT – I

- 2 (a) Define and explain the concept of random variable.
 (b) Determine whether the following is a valid distribution function or not.

$$F(x) = 1 - e^{-x/2}; \text{ for } x \geq 0$$

$$= 0; \text{ elsewhere}$$

(OR)

- 3 (a) How do you explain statistically independent events using Baye's rule?
 (b) A bag contains four balls. Two balls are drawn and are found to be white. Find the probability that all the balls are white.

UNIT – II

- 4 (a) Discuss the properties of conditional distribution function.
 (b) If the joint PDF of two dimensional random variable (x, y) is given by:

$$f_{XY}(x,y) = 2; \text{ for } 0 < x < 1, 0 < y < x$$

$$= 0; \text{ otherwise}$$

Find the marginal density function of X and Y.

(OR)

- 5 (a) Random variables X and Y have the joint density:

$$f_{XY}(x,y) = \frac{1}{24}; \text{ for } 0 < x < 6 \text{ and } 0 < y < 4$$

$$= 0; \text{ elsewhere}$$

What is the expected value of the function $g(X,Y) = (X,Y)^2$?

- (b) Briefly explain about jointly Gaussian random variables.

Contd. in page 2

UNIT – III

- 6 (a) Distinguish between ensemble average and time average of a random process.
 (b) A random process is defined as $x(t) = A \sin(\omega t + \theta)$, where A is a constant and θ is a random variable uniformly distributed over $(-\pi, \pi)$. Check $x(t)$ for stationarity.

(OR)

- 7 (a) State and prove any three properties of auto correlation function.
 (b) When do you call two random processes to be jointly wide sense stationary?

UNIT – IV

- 8 (a) Discuss the properties of cross-power density spectrum.
 (b) Find the PSD of a stationary random process for which auto correlation is $R_{xx}(\tau) = 6e^{-\alpha|\tau|}$.

(OR)

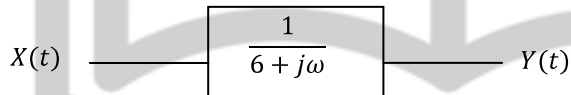
- 9 (a) If $V(f) = AT \sin \frac{(2\pi ft)}{2\pi ft}$, find the energy contained in $v(t)$.
 (b) Discuss the relationship between cross power spectrum and cross correlation function.

UNIT – V

- 10 (a) How mean value of the system response $y(t)$ is calculated?
 (b) Discuss the transmission of random process through linear system.

(OR)

- 11 (a) Discuss the following random process:
 (i) Band pass. (ii) Band limited. (iii) Narrow band.
 (b) Consider a linear system as shown below:



$X(t)$ is the input and $Y(t)$ is the output of the system. The auto correlation of $X(t)$ is $R_{xx}(\tau) = 3\delta(\tau)$. Find the PSD, auto correlation function of the output $Y(t)$.
