

**DISCRETE MATHEMATICS**

(Common to IT &amp; CSE)

Time: 3 hours

Max. Marks: 70

## PART – A

(Compulsory question)

- 1 Answer the following: (10 X 02 = 20 Marks)
- Show that the propositions  $p \rightarrow q$  and  $\neg p \vee q$  are equivalent.
  - State pigeonhole principal.
  - When a lattice is said to be bounded?
  - Prove that  $P, P \rightarrow q, q \rightarrow r \Rightarrow r$ .
  - State Lagrange's theorem in  $\delta$  group theory.
  - Prove that the identity of a subgroup is same as that of the group.
  - State any two properties of a group.
  - Find the recurrence relation satisfying the equation:  $y_n = A(3)^n + B(-4)^n$ .
  - What is the generating function of the sequence  $\{0, 1, 0-1, 0, 1, 0, -1, 0, \dots\}$
  - What is a spanning tree?

## PART – B

(Answer all five units, 5 X 10 = 50 Marks)

## UNIT - I

- 2 If  $n$  Pigeonholes are occupied by  $(kn+1)$  pigeons, where  $n$  is positive integer, prove that at least one Pigeonhole is occupied by  $k+1$  or more Pigeons. Hence, find the minimum number of  $m$  integers to be selected from  $S = \{1, 2, 3, \dots, 9\}$  so that the sum of two of the  $m$  integers are given.

(OR)

- 3 Show that  $p \vee (q \wedge r)$  and  $(p \vee q) \wedge (p \vee r)$  are logically equivalent.

## UNIT - II

- 4 In a Lattice  $(L, \leq)$ , prove that  $x \vee (y \wedge z) \leq (x \vee y) \wedge (x \vee z)$ .

(OR)

- 5 If  $(A, \leq)$  and  $(B, \leq)$  are posets, then prove that  $\{A \times B, \leq\}$  is a poset with partial order  $\leq$  defined as  $(a, b) \leq (a', b')$ , if  $a \leq a'$  in  $A$ , if  $b \leq b'$  in  $B$ .

## UNIT - III

- 6 State and prove Lagrange's theorem.

(OR)

- 7 Let  $(S, *)$  be a semi group, then prove that there exists a homomorphism  $g: S \rightarrow S^S$  Where  $\langle S^S, \circ \rangle$  is a semi group of a function from  $S$  to  $S$  under the operation of the composition.

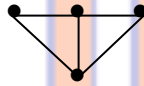
Contd. in page 2

## UNIT - IV

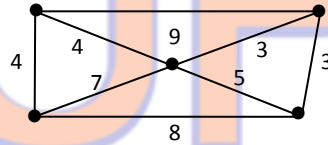
- 8 (a) Prove by mathematical induction,  $3^{2n+1} + (-1)^n 2 = 0 \pmod{3}$ .  
 (b) Using the generating function, solve the difference equation  
 $y_{n+2} - y_{n+1} - 6y_n = 0$ ,  $y_1 = 1, y_0 = 2$ .  
 (OR)
- 9 Solve the recurrence relation,  $S(n) = S(n-1) + 2(n-1)$  with  $S(0) = 3$ ,  $S(1) = 1$  by finding its generating function.

## UNIT - V

- 10 Define a planar graph, show that  $K_5$  is non-planar.  
 (OR)
- 11 (a) Define spanning tree of a graph of  $G$ . Find all the spanning trees of a following graph



- (b) Apply Kruskal's algorithm to find a minimal spanning tree of the following weighted graph.



\*\*\*\*\*