

PROBABILITY THEORY & STOCHASTIC PROCESSES

(Electronics and Communication Engineering)

Time: 3 hours

Max. Marks: 70

PART – A

(Compulsory Question)

- 1 Answer the following: (10 X 02 = 20 Marks)
- What are the conditions to be satisfied for the statistical independence of three events A, B and C?
 - Show that $P(x_1 < X \leq x_2) = F_X(x_2) - F_X(x_1)$.
 - Two random variables X and Y have the following values:
 $E[X] = E[Y] = \frac{7}{12}$, $E[XY] = \frac{1}{3}$ and $\sigma_X = \sigma_Y = \sqrt{\frac{11}{144}}$. Find the correlation coefficient.
 - Define the joint moments about the origin.
 - Define WSS random process.
 - Determine the mean-square value of a random process with autocorrelation function: $R_{XX}(\tau) = e^{-|\tau|}$.
 - A random process has the power density spectrum $S_{XX}(\omega) = \frac{6\omega^2}{1+\omega^4}$. Find the average power in the process.
 - Define rms bandwidth of the power spectrum.
 - Impulse response of a linear system is $h(t) = [1 - t][u(t) - u(t - 1)]$. The input to this system is a sample function from a random process having an autocorrelation function of $R_{XX}(\tau) = \delta(\tau)$. Find the autocorrelation of the output.
 - A stationary random process with a mean of 2 is passed through an LTI system with $h(t) = 2e^{-2t}u(t)$. Determine the mean of the output process.

PART – B

(Answer all five units, 5 X 10 = 50 Marks)

UNIT – I

- Define conditional distribution and density functions and list their properties.
 - A continuous random variable X has a PDF $f_X(x) = 3x^2$, $0 \leq x \leq 1$. Find 'a' and 'b' such that:
 (i) $P\{X \leq a\} = P\{X > a\}$. (ii) $P\{X > b\} = 0.05$.
- Define random variable and give the concept of random variable with an example.
 - The probability density function of a random variable has the form $f_X(x) = 5e^{-kx}u(x)$, where $u(x)$ is the unit step function. Find the probability that $X > 1$.

UNIT – II

- Define marginal density and distribution functions.
 - Let X and Y be jointly continuous random variables with joint probability density function:
 $f_{XY}(x, y) = x^2 + \frac{xy}{3}$; $0 \leq x \leq 1, 0 \leq y \leq 2$
 $= 0$; elsewhere
 Find: (i) $f_X(x)$. (ii) $f_Y(y)$. (iii) Are X and Y independent?

OR

- State central limit theorem for the following two cases:
 (i) Equal distributions. (ii) Unequal distributions.
 - Let $f_{XY}(x, y) = 4x + 2y$, $0 \leq x \leq 1/2$, $0 \leq y \leq 1$ and zero elsewhere. Find $P\{X \leq 1/4\}$.

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UNIT - III

- 6 (a) A random process has sample functions of the form: $X(t) = \begin{cases} A; & 0 \leq t \leq 1 \\ 0; & \text{else where} \end{cases}$
Where A is a random variable uniformly distributed from 0 to 10. Find the autocorrelation function of this process.
- (b) Show that $|R_{XX}(\tau)| \leq R_{XX}(0)$.
- OR**
- 7 (a) Show that the autocorrelation function of a stationary random process is an even function of τ .
- (b) Give the classification of random processes.

UNIT - IV

- 8 (a) A stationary random process has a two-sided spectral density given by:
 $S_{XX}(f) = \begin{cases} 10; & a < |f| < b \\ 0; & \text{else where} \end{cases}$
Find the mean-square value of the process if $a = 4$ and $b = 5$.
- (b) List the properties of power spectral density function.
- OR**
- 9 (a) For two jointly stationary random processes, the cross-correlation function is $R_{XY}(\tau) = 2e^{-2\tau}u(\tau)$. Find the two cross-spectral density function.
- (b) List the properties of cross power spectral density function.

UNIT - V

- 10 Show that $R_{YY}(\tau) = R_{XX}(\tau) * h(\tau) * h(-\tau)$.

OR

- 11 Write short notes on the following:
- (a) Bandpass random process.
- (b) Band-limited random process.
