

MATHEMATICS – I
(Common to all branches)

Time: 3 hours

Max. Marks: 70

PART – A
(Compulsory Question)

1 Answer the following: (10 X 02 = 20 Marks)

- Solve $(D^3 + 1)y = 0$.
- Solve $\frac{dy}{dx} = (x + y + 2)^2 = 0$.
- Expand e^{x+y} in a neighborhood of $(1, 1)$.
- Find the envelop of the family of curves $y = mx + m^4$ for different values of 'm'.
- Find the asymptotes of $y^3 - x^2y + 2y^2 + 4y + x$.
- Find the quadrature of the rectangular hyperbola $y = k^2/x$ from $x = a$ to $x = b$.
- $\mathcal{L}\{e^{at} \cosh bt\} =$
- $\mathcal{L}^{-1}\left\{\frac{e^{-3s}}{s^2}\right\} =$
- Prove that $\bar{a} \cdot \left(\nabla \frac{1}{r}\right) = -\frac{\bar{a} \cdot \bar{r}}{r^3}$, \bar{a} is a constant vector.
- State Green's theorem.

PART – B
(Answer all five units, 5 X 10 = 50 Marks)

UNIT - I

2 The deflection of a strut of length ℓ with one end built - in and the other end subjected to the end thrust P, satisfies $\frac{d^2y}{dx^2} + a^2y = \frac{a^2R}{P}(\ell - x)$. Find the deflection y of the strut at a distance x from the built - in end.

OR

3 Solve $(D^2 - 4D)y = e^x + \sin 3x \cos 2x$.

UNIT - II

4 Verify Maclaurin's theorem for $f(x) = (1 - x)^{5/2}$ with Lagrange form of remainder up to 3 terms with $x = 1$.

OR

5 Find the radius of curvature at any point $P(at^2, 2at)$ on the parabola $y^2 = 4ax$. Show that it is $2\frac{(SP)^{3/2}}{\sqrt{a}}$. Where S is the focus of the parabola?

UNIT - III

6 Find the volume of the solid generated by revolution of the loop of the curve $y^2(a - x) = x^2(a + x)$ about the $x -$ axis.

OR

7 Evaluate the integral $\int_{y=0}^1 \int_{x=y}^a \frac{xdxdy}{x^2+y^2}$.

UNIT - IV

8 Find the Laplace transform for $f(t) = \left(\sqrt{t} - \frac{1}{\sqrt{t}}\right)^3$.

OR

9 The triangular wave function defined by $f(t) = \begin{cases} t, & 0 < t < a \\ 2a - t, & a < t < 2a \end{cases}$ and $f(t + 2a) = f(t)$. Find Laplace transform of $f(t)$.

UNIT - V

10 Find the directional derivative of $\phi(x, y, z) = xy + yz + zx$ in the direction of $-\sqrt{2}\bar{i} + \bar{j} + 2\bar{k}$ at the point $(1, 2, 0)$.

OR

11 If $\bar{F} = 2xz\bar{i} - x\bar{j} + y^2\bar{k}$ evaluate $\iiint_V \bar{F} \cdot d\mathbf{v}$ where V is the region bounded by the surface $x = 0, y = 0, x = 2, y = 6, z = x^4, z = 4$.
