

PROBABILITY THEORY & STOCHASTIC PROCESSES

(Electronics and Communication Engineering)

Time: 3 hours

Max. Marks: 70

PART – A

(Compulsory Question)

- 1 Answer the following: (10 X 02 = 20 Marks)
- State Baye's theorem.
 - Three coins are tossed in succession. Find out the probabilities of occurrence of two consecutive heads.
 - State central limit theorem.
 - Find the expected value of the face value while rolling fair die?
 - Define cross-covariance function.
 - Give any two examples for poisson random process.
 - A random process has the power density spectrum $S_{XX}(\omega) = \frac{6\omega^2}{1+\omega^4}$. Find the average power in the process.
 - What is power spectral density? Mention its importance.
 - Define the following random process: (i) Band limited. (ii) Narrow band.
 - What are the two conditions that are to be satisfied by the power spectrum $\frac{\omega^2}{\omega^6+3\omega^2+3}$ to be a valid power density spectrum?

PART – B

(Answer all five units, 5 X 10 = 50 Marks)

UNIT - I

- 2 (a) A pack contains 4 white and 2 green pencils, another contains 3 white and 5 green pencils. If one pencil is drawn from each pack, find the probability that (i) Both are white. (ii) One is white and another is green
- (b) Explain about joint and conditional probability.
- OR
- 3 (a) Consider the experiment of tossing four fair coins. The random variable X is associated with the number of tails showing. Compute and sketch the CDF of X.
- (b) Define probability density function. List its properties.

UNIT - II

- 4 (a) Let X and Y be jointly continuous random variables with joint density function

$$f_{XY}(x,y) = \begin{cases} xy e^{-\left(\frac{x^2+y^2}{2}\right)}; & \text{for } x>0, y>0 \\ 0; & \text{otherwise} \end{cases}$$

(i) Check whether x and y are independent.

(ii) Find P (x≤1, y≤1).

- (b) How expectation is calculated for two random variables?

OR

- 5 (a) Prove the following:
 $\text{Var}(ax+by) = a^2 \text{var}(x) + b^2 \text{var}(y) + 2ab \text{cov}(x,y)$
- (b) Explain central limit theorem.

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UNIT - III

- 6 (a) Explain about mean-ergodic process.
 (b) If $x(t)$ is a stationary random process having mean = 3 and auto correlation function:
 $R_{xx}(\tau) = 9 + 2e^{-|\tau|}$. Find the mean and variance of the random variable.

OR

- 7 (a) Explain the significance of auto correlation.
 (b) Find auto correlation function of a random process whose power spectral density is given by $\frac{4}{1+\frac{\omega^2}{4}}$

UNIT - IV

- 8 (a) Briefly explain the concept of cross power density spectrum.
 (b) Find the cross correlation of functions $\sin \omega t$ and $\cos \omega t$.

OR

- 9 (a) The power spectral density of a stationary random process is given by

$$S_{xx}(\omega) = \begin{cases} A; & -k < \omega < k \\ 0; & \text{otherwise} \end{cases}$$

Find the auto correlation function.

- (b) Discuss the properties of power spectral density.

UNIT - V

- 10 (a) A Gaussian random process $X(t)$ is applied to a stable linear filter. Show that the random process $Y(t)$ developed at the output of the filter is also Gaussian.
 (b) Discuss about cross correlation between the input $X(t)$ and output $Y(t)$.

OR

- 11 (a) Derive the relation between PSDs of input and output random process of an LTI system.
 (b) The input voltage to an RLC series circuit is a stationary random process $X(t)$ with $E[X(t)] = 2$ and $R_{xx}(\tau) = 4 + \exp(-2|\tau|)$. Let $Y(t)$ is the voltage across capacitor. Find $E[Y(t)]$.
