

Code No: 133BC

R16

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech II Year I Semester Examinations, April/May - 2018

MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

(Common to CSE, IT)

Time: 3 Hours

Max. Marks: 75

Note: This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A.

Part B consists of 5 Units. Answer any one full question from each unit.

Each question carries 10 marks and may have a, b, c as sub questions.

PART- A

(25 Marks)

- 1.a) Construct the truth table for the following formula:  
 $\neg(P \vee (Q \wedge R)) \leftrightarrow ((P \vee Q) \wedge (P \vee R))$  [2]
- b) Explain duality law. [3]
- c) Give the formal definition for the composition of binary relations. [2]
- d) What are the properties of a group? [3]
- e) State addition principle and give an example of a problem solved by addition principle. [2]
- f) State pigeon-hole principle. [3]
- g) What is the general form of a first-order recurrence relation? [2]
- h) What is the generating function of 1, -1, 1, -1, ... [3]
- i) If a simple graph  $G$  contains  $n$  vertices and  $m$  edges, how many number of edges are present in Graph  $G'$  (complement of  $G$ ). [2]
- j) How many edges are present in a complete graph with  $n$  vertices? Explain. [3]

PART- B

(50 Marks)

- 2.a) Show the following equivalence without constructing the truth table.  
 $((P \wedge Q \wedge A) \rightarrow C) \wedge (A \rightarrow (P \vee Q \vee C)) \leftrightarrow (A \wedge (P \leftrightarrow Q)) \rightarrow C$
- b) Without constructing a truth table, show that  $A \wedge E$  is not a valid consequence of  
 $A \leftrightarrow B, B \leftrightarrow (C \wedge D), C \leftrightarrow (A \vee E), A \vee E$  [5+5]

OR

- 3.a) Obtain the principal disjunctive and conjunctive normal form of the following formula.  
 $(P \rightarrow (Q \wedge R)) \wedge (\neg P \rightarrow (\neg Q \wedge \neg R))$
- b) For the following formulas, let the universe be  $\mathbb{R}$ . Translate each of the following sentences into a formula (using quantifiers):
  - i) There is a smallest number.
  - ii) Every positive number has a square root. (Do not use the square root symbol; use only multiplication.) [5+5]

4.a) Consider the following Hasse diagram of a partially ordered set  $\langle P, R \rangle$ , where  $P = \{x_1, x_2, x_3, x_4, x_5\}$ . Find the least and greatest members in  $P$  if they exist. Also find the maximal and minimal elements of  $P$ . Find the upper and lower bounds of  $\{x_2, x_3, x_4\}$ ,  $\{x_2, x_4, x_5\}$  and  $\{x_1, x_2, x_3\}$ . Also indicate the LUB and GLB of these subsets if they exist.

b) Let  $n \in \mathbb{N}^+$  and  $G_1, G_2, \dots, G_n$  be groups, and consider

$$\prod_{i=1}^n G_i := G_1 \times G_2 \times \dots \times G_n = \{(a_1, a_2, \dots, a_n) : a_i \in G_i, \forall i = 1, 2, \dots, n\}$$
 with the operation  $\dagger$

where if  $x = (a_1, a_2, \dots, a_n)$  and  $y = (b_1, b_2, \dots, b_n)$ , then  $x \dagger y = (a_1 b_1, a_2 b_2, \dots, a_n b_n)$ , where each product  $a_i b_i$  is performed according to the operation of the group  $G_i$ . Show

that  $\prod_{i=1}^n G_i$  is a group. [5+5]

OR

5.a) Find the transitive closure of the relation  $R = \{(1,2), (2,3), (3,4), (4,1)\}$ . Show  $R^i$  for all values of  $i$  that give new elements of the transitive closure.

b) Find all the subgroups of (i)  $(\mathbb{Z}_{12}, +_{12})$ ; and (ii)  $(\mathbb{Z}_7, \times_7)$ . [5+5]

6. In the United States and Canada, a telephone number is a 10-digit number of the form  $NXX - NXX - XXXX$  where  $N \in \{2, 3, \dots, 9\}$  and  $X \in \{0, 1, 2, \dots, 9\}$ . How many telephone numbers are possible? The first three digits of a telephone number are called an area code. How many different area codes must a city with 23,000,000 phones have? A previous scheme for forming a telephone numbers required a format of  $NYX - NXX - XXXX$  where  $N$  and  $X$  are defined as above and  $Y$  is either a 0 or a 1. How many more phone numbers are possible under the new format than under the old format? [10]

OR

7.a) How many four letter words can be formed using the letters  $a, a, a, b, b, c, c, c, c, d, d$ ?

b) Expand  $(2x - y)^7$  using the Binomial Theorem. [5+5]

8.a) Solve the recurrence relation  $a_n = 2a_{n-1} + 3a_{n-2}$  for  $n \geq 2$  where  $a_0 = 2$  and  $a_1 = 2$ .

b) Using generating function find  $a_n$  in terms of  $n$  if  $a_0 = 1, a_1 = 2$  and  $a_{n+2} = 5a_{n+1} - 4a_n$  for  $n \geq 0$ . [5+5]

OR

9.a) Solve the recurrence relation  $T(n) = 4T(n-1) + 2^n$ , with  $T(0) = 6$ .

b) Find the coefficient of  $x^{2005}$  in the generating function  $\frac{1}{(1+5x)^2}$ . [5+5]

