

Time: 3 Hours

Max. Marks: 75

Note: This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A.

Part B consists of 5 Units. Answer any one full question from each unit.

Each question carries 10 marks and may have a, b, c as sub questions.

## Part- A

(25 Marks)

- 1.a) Find  $n$ , if  $\vec{f} = r^n \vec{r}$  is solenoidal. [2M]
- b) Find the angle between the surfaces  $x^2 + y^2 + z^2 = 9$  and  $z = x^2 + y^2 - 3$  at the point  $(2, 1, 2)$ . [3M]
- c) If  $f(x) = x^4$  in  $(-1, 1)$  then find the Fourier coefficient  $b_n$ . [2M]
- d) Write any three properties of Fourier transforms. [3M]
- e) Evaluate  $\Delta^{10}[(1-ax)(1-bx^2)(1-cx^3)(1-dx^4)]$ . [2M]
- f) Write the normal equations to fit a curve  $y = ae^{bx}$  for the given data by the method of least squares. [3M]
- g) Define transcendental equation and give an example. [2M]
- h) Write short notes on iteration method to find a root for  $f(x) = 0$ . [3M]
- i) Define initial value problem and give an example. [2M]
- j) Write the finite difference formula for  $y'(x)$  and  $y''(x)$ . [3M]

## Part- B

(50 Marks)

- 2.a) With usual notations of vector calculus, prove that  $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$ .
- b) Apply Green's theorem to evaluate  $\oint_C (xy + y^2) dx + x^2 dy$ , where  $C$  is the boundary of the area enclosed by the  $x$ -axis and the upper half of the circle  $x^2 + y^2 = a^2$ .

OR

- 3.a) Find the work done by  $\vec{F} = (2x - y - z)\vec{i} + (x + y - z)\vec{j} + (3x - 2y - 5z)\vec{k}$  along a curve  $C$  in the  $xy$ -plane given by  $x^2 + y^2 = 9$ ,  $z = 0$ .
- b) Use Gauss divergence theorem, to evaluate  $\iiint_S (x dy dz + y dz dx + z dx dy)$ , where  $S$  is the portion of the plane  $x + 2y + 3z = 6$ , which lies in the first octant.

- 4.a) Expand  $f(x) = x \sin x$  as a Fourier series in the interval  $0 < x < 2\pi$ .
- b) Express  $f(x) = \begin{cases} 1, & \text{for } 0 \leq x \leq \pi \\ 0, & \text{for } x > \pi \end{cases}$ , as a Fourier sine integral and hence evaluate  $\int_0^{\infty} \frac{1 - \cos(\pi\lambda)}{\lambda} \sin(x\lambda) d\lambda$ .

OR

- 5.a) Find the half-range cosine series for  $f(x) = x^2$  in the range  $0 \leq x \leq \pi$ .
- b) Find the Fourier sine transform of  $\frac{e^{-ax}}{x}$ .

- 6.a) If  $P$  is the pull required to lift a load  $W$  by means of a pulley block, find a linear law of the form  $P = mW + c$  connecting  $P$  and  $W$ , using the following data:

$P=$	12	15	21	25
$W=$	50	70	100	120

- where  $P$  and  $W$  are taken in  $kg.wt$ . Compute  $P$  when  $W = 150 kg.wt$ .  
 b) Find the missing values in the following data

$x:$	45	50	55	60	65
$y:$	3.0	-	2.0	-	-2.4

OR

- 7.a) The pressure  $p$  of wind corresponding to velocity  $v$  is given by the following data. Estimate  $p$  when  $v = 25$ .

$v:$  10 20 30 40

$p:$  1.1 2 4.4 7.9

- b) Find the polynomial  $f(x)$  by using Lagrange's formula and hence find  $f(3)$  for  
 $x:$  0 1 2 5  
 $f(x):$  2 3 12 147.

- 8.a) Using Newton's iterative method, find the real root of  $x \log_{10} x = 1.2$  correct to four decimal places.

- b) Solve:  $20x + y - 2z = 17$ ,  $3x + 20y - z = -18$ ,  $2x - 3y + 20z = 25$  by Gauss-Seidel iterative method.

OR

- 9.a) Solve by L-U decomposition method:  $x + y + z = 9$ ,  $2x - 3y + 4z = 13$ ,  $3x + 4y + 5z = 40$ .

- b) Use the method of false position to find the fourth root of 32 correct to three decimal places.

- 10.a) Using Runge-Kutta method of order 4, find  $y(0.2)$  for the equation  $\frac{dy}{dx} = \frac{y-x}{y+x}$ ,  $y(0) = 1$ . Take  $h = 0.2$ .

- b) A solid of revolution is formed by rotating about the  $x$ -axis, the area between the  $x$ -axis, the lines  $x = 0$  and  $x = 1$  and a curve through the points with the following co-ordinates

$x:$  0.00 0.25 0.50 0.75 1.00

$y:$  1.0000 0.9896 0.9589 0.9089 0.8415.

Estimate the volume of the solid formed using Simpson's rule.

OR

11. Use power method to find the numerically largest Eigen value and the

corresponding Eigen vector of  $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ . Find also the least Eigen value

and hence the third Eigen value also.