

R16

Code No: 132AB

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech I Year II Semester Examinations, April - 2018

MATHEMATICS-II

(Common to EEE, ECE, CSE, EIE, IT, ETM)

Time: 3 hours

Max. Marks: 75

Note: This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A.

Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART-A

(25 Marks)

1.a) Find $L\{t^2 u(t-1)\}$. [2]

b) Obtain the inverse Laplace transform of $F(s) = \cot^{-1} s$. [3]

c) Find the value of $\Gamma\left(\frac{-1}{2}\right)$. [2]

d) Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{\tan \theta} d\theta$ using Beta and Gamma functions. [3]

e) Evaluate $\iint_R \sqrt{x^2 + y^2} dx dy$ by changing to polar coordinates, where R is the region in the xy -plane bounded by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$. [2]

f) Find the value of the triple integral $\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{4}} \int_0^1 r^2 dr d\theta d\phi$. [3]

g) Find the normal vector and unit normal vector to the surface $z^2 = x^2 - y^2$ at $(2, 1, \sqrt{3})$. [2]

h) If \vec{a} is a constant vector and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, prove that $\text{div}(\vec{a} \times \vec{r}) = 0$. [3]

i) Evaluate $\int_c (x^2 + yz) dz$, where c is given by $x=t, y=t^2, z=3t, 1 \leq t \leq 2$. [2]

j) State Gauss's divergence theorem. [3]

PART-B

(50 Marks)

2.a) Find the Laplace transform of $f(t) = e^{-t} \left[\int_0^t \frac{\sin u}{u} du \right]$.

b) Find the Laplace transform of the periodic function $f(t) = t, 0 \leq t \leq a, f(t+a) = f(t)$.

[5+5]

OR

3.a) Obtain $L^{-1} \left\{ \frac{s}{(s^2+1)^2} \right\}$ using convolution theorem.

b) Solve the initial value problem $y''+3y'+2y=e^{-t}$, $y(0)=0$, $y'(0)=-1$ using Laplace transforms. [5+5]

4.a) Evaluate $\int_0^{\infty} x^4 e^{-2x^2} dx$.

b) Prove that $\beta(m,n) = \int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx$. [5+5]

OR

5.a) Prove that $\beta(m,n) = 2 \int_0^{\pi/2} \sin^{2m-1} \theta \cos^{2n-1} \theta d\theta = \beta(m+1,n) + \beta(m,n+1)$.

b) Prove that $\int_0^1 x^m (\ln x)^n dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$ where $m > 1$ and n is a positive integer. [5+5]

6.a) Change the order of integration in $\int_0^1 \int_{x^2}^{2-x} xy dx dy$ and hence evaluate the same.

b) Find the area of the region bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$. [5+5]

OR

7.a) Evaluate $\iiint_v (1-x) dx dy dz$, where v is the space in the first octant below the plane $3x+2y+z=6$.

b) Find the volume of the solid enclosed between the surfaces $x^2+y^2=4$ and $x^2+z^2=4$. [5+5]

8.a) Find the values of a and b so that the surface $ax^2 - byz = a+2$ is orthogonal to the surface $4x^2y+z^3=4$ at $(1, -1, 2)$.

b) Find the directional derivative of the scalar function $f(x,y,z) = xyz$ at $(1, 4, 9)$ in the direction of the line from $(1, 2, 3)$ to $(1, -1, -3)$. [5+5]

OR

9.a) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$, show that $\nabla \cdot \left(\frac{\vec{r}}{r^3} \right) = 0$.

b) Prove that $\text{curl}(\text{curl } \vec{v}) = \nabla(\nabla \cdot \vec{v}) - \nabla^2 \vec{v}$. [5+5]

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10.a) Show that $\int_c (2xy+3)dx + (x^2-4z)dy - 4ydz$, where c is any path joining $(0, 0, 0)$ to $(1, -1, 3)$, does not depend on the path c and evaluate the integral.

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b) Apply Stoke's theorem to evaluate $\oint (x+y)dx + (2x-z)dy + (y+z)dz$, where c is the boundary of the triangle with vertices $(-2, 0, 0)$, $(0, 3, 0)$ and $(0, 0, 6)$. [5+5]

OR
11. Verify Green's theorem for $\oint_c e^{-x}(\cos y dx - \sin y dy)$, where c is the rectangle with vertices $(0,0)$, $(\pi, 0)$, $(\pi, \frac{\pi}{2})$ and $(0, \frac{\pi}{2})$. [10]

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