

Code No: 111AB

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech I Year Examinations, October/November - 2016

MATHEMATICS-I

(Common to all Branches)

Time: 3 hours

Max. Marks: 75

**Note:** This question paper contains two parts A and B. Part A is compulsory which carries 25 marks. Answer all questions in Part A. Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

## PART- A

(25 Marks)

- 1.a) Define Eigen vector of a matrix. [2]
- b) Write the working procedure to solve the system of non-homogenous equations. [3]
- c) Verify for  $x = u$ ,  $y = u \tan v$ ,  $z = w$ ,  $J \left( \frac{x, y, z}{u, v, w} \right) \times J' \left( \frac{u, v, w}{x, y, z} \right) = 1$ . [2]
- d) Give an example of a function that is continuous on  $[-1, 1]$  and for which mean value theorem does not hold, explain. [3]
- e) Show that  $\beta(p, q) = \beta(p+1, q) + \beta(p, q+1)$ . [2]
- f) Evaluate  $\int_0^1 \int_1^{2-x} xy dx dy$ . [3]
- g) Explain the method of solving Bernoulli equation. [2]
- h) Solve  $(D^4 + 2D^2n^2 + n^4)y = 0$ . [3]
- i) State and prove change of scale property of Laplace transforms. [2]
- j) Prove that  $L^{-1}\{F(s)\} = f(t)$  and  $f(0) = 0$  then  $L^{-1}\{sF(s)\} = \frac{df}{dt}$ . [3]

## PART-B

(50 Marks)

2. Determine a non-singular matrix P such that  $P^TAP$  is a diagonal matrix, where

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 2 & 3 & 0 \end{bmatrix} \quad [10]$$

OR

- 3.a) Show that the two matrices A,  $C^{-1}AC$  have the same latent roots.

- b) For a matrix  $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$  find the Eigen values of  $3A^3 + 5A^2 - 6A + 2I$ . [5+5]

- 4.a) Find the minimum and maximum values of  $\sin x + \sin y + \sin(x+y)$ .

- b) If  $u = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$ ,  $x^2 + y^2 + z^2 \neq 0$  then evaluate  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$ . [5+5]

OR

5.a) Prove that  $\frac{\pi}{6} + \frac{1}{5\sqrt{3}} < \sin^{-1}\left(\frac{3}{5}\right) \leq \frac{\pi}{6} + \frac{1}{8}$ .

b) Verify Lagrange's mean value theorem for  $f(x) = \begin{cases} x \sin \frac{1}{x} & (x \neq 0) \\ 0 & (x = 0) \end{cases}$  in  $[-1, 1]$ . [5+5]

6.a) Evaluate  $\iiint_V x^{l-1} y^{m-1} z^{n-1} dx dy dz$  where  $V$  is the region  $x \geq 0, y \geq 0, z \geq 0$  and the plane  $x + y + z < 1$ .

b) Express the integral  $\int_0^\infty \frac{x^c}{c^x} dx (c > 1)$  in terms of Gamma function. [5+5]

OR

7.a) By changing the order of integration and evaluate  $\int_0^b \int_0^{\frac{a\sqrt{b^2-y^2}}{b}} xy dy dx$ .

b) Find the area enclosed by the parabolas  $x^2 = y$  and  $y^2 = x$ . [5+5]

8.a) The number  $N$  of bacteria in a culture grows at a rate proportional to  $N$ . The value of  $N$  was initially 100 and increased to 332 in one hour. What was the value of  $N$  after  $1\frac{1}{2}$  hour?

b) Solve  $(x - y)dx - dy = 0, y(0) = 2$ . [5+5]

OR

9. Solve  $(D^2 - 4D + 4)y = x^2 \sin x + e^{2x} + 3$ . [10]

10.a) State and prove convolution theorem for Laplace transforms.

b) Find the Laplace transform of  $f(t) = |t - 1| + |t + 1|, t \geq 0$ . [5+5]

OR

11.a) Solve the differential equation using Laplace transforms

$$\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2x = e^{-t}; x(0) = 0, x'(0) = 1.$$

b) Evaluate  $L\left\{\int_0^t e^{-t} \cos t dt\right\}$ . [5+5]

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