

I B. Tech II Semester Supplementary Examinations, April/May - 2018
MATHEMATICS-III
 (Com. to All Branches)

Time: 3 hours

Max. Marks: 70

- Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B**)
 2. Answering the question in **Part-A** is Compulsory
 3. Answer any **THREE** Questions from **Part-B**

PART -A

1. a) Reduce the matrix $A = \begin{bmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}$ to its normal form and hence find the rank. (4M)
- b) Prove that $|A| / \lambda$ is an Eigen value of the matrix $\text{adj } A$. (3M)
- c) Find the asymptotes in the curve $y^2(a+x) = x^2(3a-x)$. (3M)
- d) Prove that $B(m, n) = B(m+1, n) + B(m, n+1)$. (4M)
- e) Find the unit normal vector to the surface $\varphi(x, y, z) = x^2 + y^2 + z^2$ at $(-1, -1, -2)$ (4M)
- f) Evaluate $\int f \cdot dr$ where $f = (2xy + 3z^3)i + x^2j + 3xz^2k$ along the straight line joining $(0,0,0)$ and $(2,1,2)$. (4M)

PART -B

2. a) Show that the only real value of λ for which the following equations have non-trivial solution is 6 and solve them, when $\lambda=6$. $x+2y+3z=\lambda x$; $3x+y+2z=\lambda y$; $2x+3y+z=\lambda x$. (8M)
- b) Solve the equations $5x + y + z + w = 4$, $x + 7y + z + w = 12$, $x + y + 6z + w = -5$, $x + y + z + 4w = -6$ by Gauss-seidal method. (8M)
3. a) Reduce the quadratic form $x^2+y^2+2z^2-2xy+4xz+4yz$ to the canonical form and find the rank, index and signature. (8M)

- b) Verify Cayley-Hamilton for the matrix $A = \begin{bmatrix} 8 & -8 & 2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ find A^{-1} (8M)

4. a) Find the perimeter of the Loop of the curve $3ay^2=x(x-a)^2$. (8M)
- b) Change the order of the Integration in $\int_0^1 \int_{x^2}^{2-x} xy \, dx \, dy$ and hence evaluate the double integral. (8M)
5. a) Prove that $\Gamma(n)\Gamma(1-n) = \frac{\pi}{\sin n\pi}$ (8M)
- b) Prove that $\int_0^1 \frac{dx}{\sqrt{1-x^n}} = \frac{\sqrt{\pi}}{n} \frac{\Gamma\left(\frac{1}{n}\right)}{\Gamma\left(\frac{1}{n} + \frac{1}{2}\right)}$ (8M)
6. a) Find the directional derivative of $\phi(x, y, z) = x^4 + y^4 + z^4$ at the point $(-1, 2, 3)$ in the direction towards the point $(2, -1, -1)$. (8M)
- b) If ϕ be two scalar point functions and \vec{f} be two vector point functions then show that $\nabla \cdot (\phi \vec{f}) = \nabla \phi \cdot \vec{f} + \phi (\nabla \cdot \vec{f})$ (8M)
7. a) Verify Gauss divergence theorem $F = (x^3 - yz)i - 2x^2yj + zk$ over the surface of the cube bounded by $x = y = z = a$. (8M)
- b) Evaluate $\int_c (xy - y^2)dx + x^2ydy$ along the closed curve formed by $y = 0$, $x = 1$ and $y = x$ by greens theorem. (8M)