Code No: RT22042 **R13** SET - 1

II B. Tech II Semester Regular Examinations, April/May - 2016
RANDOM VARIABLES AND STOCHASTIC PROCESSES
(Electronics and Communications Engineering)

Time: 3 hours Max. Marks: 70

Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B)**
2. Answer **ALL** the question in **Part-A**
3. Answer any **THREE** Questions from **Part-B**

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PART –A

1. a) Define probability mass function and list its properties. (3M)
   b) Show that the first central moment is zero. (4M)
   c) Define central limit theorem. (4M)
   d) Distinguish between deterministic and non-deterministic random processes. (3M)
   e) Show that $S_{XX}(-\omega) = S_{XX}(\omega)$. (4M)
   f) A WSS random process $X(t)$ is applied to the input of an LTI system with transfer function $H(\omega) = \frac{3}{2+j\omega}$. Find the mean of the output $Y(t)$ of the system if $E[X(t)] = 2$. (4M)

PART -B

2. a) Two dice are thrown. The square of the sum of the points appearing on the two dice is a random variable X. Determine the values taken by X, and the corresponding probabilities. (8M)
   b) State and prove the properties of probability density function. (8M)

3. a) Let $Y = 2X + 3$. If the random variable X is uniformly distributed over [-1, 2], determine $f_Y(y)$. (8M)
   b) Find the second central moment of a random variable with PDF $f_X(x) = ae^{-ax}u(x)$ (8M)

4. a) State central limit theorem for the following cases: (8M)
   i) Equal distributions ii) Unequal distributions
   b) Determine $f_Z(z)$ in terms of $f_X(x)$and $f_Y(y)$, if $Z = X + Y$. (8M)

5. a) Give the classification of random processes. (8M)
   b) A random process is given by $X(t) = A\cos(\omega_c t + \Theta)$, where$\omega_c$ is a constant and A and Θ are independent random variables uniformly distributed in the ranges (-1, 1) and (0, 2 π), respectively. Determine$R_{XX}(t_1, t_2)$. (8M)

6. a) For each of the following functions, state whether it can be valid PSD of a real random process: i) $\frac{(2\pi f)^2}{(2\pi f)^2+16}$ ii) $j[\delta(f + f_o) + \delta(f - f_o)]$ (8M)
   b) State and prove the properties of power spectral density. (8M)

7. a) Let $Y(t)$ be the output of an LTI system with impulse response $h(t)$. Find the cross-correlation between the input $X(t)$ and output $Y(t)$. (8M)
   b) Write notes on the following terms: i) Thermal noise ii) Narrowband noise (8M)

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PART –A

1. a) Define mixed random variable and give an example. (3M)
   b) The random variable X takes the values 0 and 1 with probabilities α and β respectively. Find the mean of X. (4M)
   c) List any three properties of jointly Gaussian random variables. (3M)
   d) Show that $|R_{XX}(\tau)| \leq R_{XX}(0)$. (4M)
   e) Show that $S_{XY}(\omega) = S^*_{YX}(\omega)$. (4M)
   f) Define generalized Nyquist theorem. (4M)

PART –B

2. a) Distinguish between discrete, continuous and mixed random variables with suitable examples. (8M)
   b) A binary source generates digits 1 and 0 randomly with probabilities 0.6 and 0.4, respectively. What is the probability that two 1s and three 0s will occur in a five-digit sequence. Hint: Let X be the random variable denoting the number of 1s generated in a five-digit sequence. (8M)

3. a) Let $Y = X^2$. Find $f_Y(y)$, if $X = N(0;1)$. (8M)
   b) Define characteristic function and list its properties. (8M)

4. a) If X and Y are independent, then show that $E[XY] = E[X]E[Y]$. (8M)
   b) Let X and Y be defined by $X = cos\Theta$ and $Y = sin\Theta$, where Θ is a random variable uniformly distributed over $[0,2\pi]$. Show that X and Y are not independent. (8M)

5. a) Show that for a WSS process $X(t)$, $R_{XX}(0) \geq |R_{XX}(\tau)|$. (8M)
   b) Given a random process $X(t) = kt$, where k is a random variable uniformly distributed in the range (-1, 1). Is the process ergodic? (8M)

6. a) Show that the power spectrum of a real random process $X(t)$ is real. (8M)
   b) Define cross power spectral densities and list all the properties cross PSDs. (8M)

7. a) Suppose that the input to a differentiator is the WSS random process. Determine the power spectral density of output. (8M)
   b) Derive the expression for noise figure of two-stage cascaded network. (8M)
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Code No: RT22042 **R13** SET - 3

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PART –A

1. a) What are the conditions for a function to be a random variable? (4M)

b) Find the relationship between $f_X(x)$ and $f_Y(y)$ if $Y = aX + b$. (4M)

c) Define marginal probability density functions. (3M)

d) Define : i)covariance-stationary random process (4M)
ii) Autocorrelation-stationary random process

e) If $R_{YY}(\tau) = R_{XX}(\tau)\cos(\omega_c\tau)$, determine $S_{YY}(\omega)$. (4M)

f) List the properties of narrowband random process. (3M)

PART –B

2. a) The PDF of a random variable is given by $f_X(x) = ke^{-ax}u(x)$, where a is a positive constant. Determine the value of constant k. (8M)

b) A noisy transmission channel has a per-digit error probability $p_e = 0.001$. Determine the probability of more than one error in 100 received digits using Poisson approximation. (8M)

3. a) Let $Y = aX + b$. Find the PDF of Y, if $X = N(\mu; \sigma^2)$. (8M)

b) State and prove Chebychev's inequality. (8M)

4. a) Let Z is the sum of two independent random variables X and Y. Find the PDF of Z. (8M)

b) List all the properties of jointly Gaussian random variables. (8M)

5. a) Sketch the ensemble of the random process $X(t) = Acos(\omega_c t + \Theta)$, where ω_c and Θ are constants and A is a random variable uniformly distributed in the range $(-A, A)$. Just by observing the ensemble, determine whether this is a stationary or a non-stationary process. (8M)

b) List all the properties of auto-correlation function. (8M)

6. State and prove Wiener-Khinchin relation. (16M)

7. a) Derive the relationship between autocorrelation of output random process of an LTI system when the input is a WSS process. (8M)

b) Describe the method of modeling a thermal noise source. (8M)
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PART –A

1. a) A noisy transmission channel has a per-digit error probability $p_e = 0.01$.. Calculate the probability of more than one error in 10 received digits. (4M)
b) Determine the mean value of uniform random variable. (4M)
c) When the two random variables X and Y are said to jointly Gaussian.? (3M)
d) Autocorrelation of a random process $X(t)$ is given by $\frac{A^2}{2}\cos(\omega\tau)$. Determine the mean-square value of $X(t)$. (4M)
e) If $R_{XX}(\tau) = A^2 e^{-2\alpha|\tau|}$, determine $S_{XX}(\omega)$. (4M)
f) Draw the power spectrum of (3M)
i) White noise ii) Band-limited white noise

PART –B

2. a) Define the conditional density and distribution functions. List all the properties of conditional density and distribution functions. (8M)
b) In an experiment, a trial consists of two successive tosses of a fair coin. If a random variable X takes the number of tails appearing in a trial, determine the CDF of X. (8M)

3. a) Write notes on monotonic transformations for a continuous random variable. (8M)
b) Show that $E[X+Y] = E[X] + E[Y]$. (8M)

4. a) The joint PDF of two continuous random variables is given by (8M)
$$f_{XY}(x,y) = xy\, e^{-x^2} . e^{\frac{-y^2}{2u(x)u(y)}}$$
Are X and Y independent?
b) Write notes on linear transformation of Gaussian random variables. (8M)

5. a) A random process is given by $X(t) = at + b$, where b is a constant and a is an r.v uniformly distributed in the range (-2, 2). Is the process WSS? (8M)
b) Derive an expression that relates autocorrelation function and auto covariance function. (8M)

6. Show that the autocorrelation function and power spectral density forms Fourier transform pair. (16M)

7. Write notes on the following:
a) Band limited white noise (8M)
b) Thermal noise (8M)

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