

II B. Tech II Semester Supplementary Examinations, Nov/Dec-2016
RANDOM VARIABLES AND STOCHASTIC PROCESSES
 (Electronics and Communications Engineering)

Time: 3 hours

Max. Marks: 70

- Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B**)
 2. Answer **ALL** the question in **Part-A**
 3. Answer any **THREE** Questions from **Part-B**

PART-A

1. a) What is a Random variable? Explain different types of Random variable
 b) What is Transformation? Classify the different types Transformation of Random Variable
 c) Write properties of Joint Density Function
 d) Write the properties of Autocorrelation Function of Random Process
 e) Write the properties of power density spectrum
 f) A white noise $X(t)$ of psd $N_0/2$ is applied on an LTI system having impulse response $h(t)$.
 If $Y(t)$ is the output find $E[Y^2(t)]$

PART-B

2. a) A random current is described by the sample space. A random variable X is defined by

$$X(i) = \begin{cases} -2 & i \leq -2 \\ i & -2 < i \leq 1 \\ 1 & 1 < i \leq 4 \\ 6 & 4 < i \end{cases}$$

Show, by a sketch, the value X into which the values of i are mapped by X .

What type of random variable is X ?

- b) Explain Gaussian random variable with neat sketches?
3. a) A random variable X can have values -4, -1, 2, 3, and 4, each with probability 0.2. Find
 (i) the density function (ii) the mean (iii) the variance of the random variable $Y = X^2$.
 b) Find the expected value of the function $g(X) = X^3$ where X is a random variable defined by the density

$$f_X(x) = \left(\frac{1}{2}\right) u(x) \exp(-x/2).$$

4. a) Define random variables V and W by

i) $V = X + aY$

ii) $W = X - aY$

Where a is real number and X and Y random variables, Determine a in terms of X and Y such V and W are orthogonal?

- b) Gaussian random variables X and Y have first and second order moments $m_{10} = -1.1$, $m_{20} = 1.16$, $m_{01} = 1.5$, $m_{02} = 2.89$, $R_{XY} = -1.724$. Find C_{XY} , ρ ?

5. a) Let $X(t)$ be a stationary continuous random process that is differentiable. Denote its time

derivative by $\dot{X}(t)$. Show that $E\left[\dot{X}(t)\right] = 0$.

- b) A random process is defined by $X(t) = A$, where A is a continuous random variable uniformly distributed on (0, 1). Determine the form of the sample functions, classify the process

6. a) Derive the relationship between cross-power spectrum and cross-correlation

- b) A random process is given by $X(t) = A \cos(\Omega t + \theta)$ where A is a real constant, Ω is a random variable with density function $f_{\Omega}(\Omega)$ and θ is a random variable uniformly distributed over the interval (0, 2π) independent of Ω . Show that the power spectrum of $X(t)$ is $S_{XX}(\omega) = \frac{\pi A^2}{2} [f_{\Omega}(\omega) + f_{\Omega}(-\omega)]$ and also find P_{YY} .

7. A random noise $X(t)$, having a power spectrum

$$S_{XX}(\omega) = \frac{3}{49 + \omega^2}$$

is applied to a differentiator that a transform function $H_1(\omega) = j\omega$, the differentiator's output is applied to a network for which $h_2(t) = u(t)t^2 \exp(-7t)$ and the network's response is a noise denoted by $Y(t)$. Find the following

- (a) What is the average power in $X(t)$
(b) Find the power spectrum of $Y(t)$
