Code: 15A04304

R15

B.Tech II Year I Semester (R15) Supplementary Examinations June 2017

PROBABILITY THEORY & STOCHASTIC PROCESSES

(Electronics & Communication Engineering)

Time: 3 hours Max. Marks: 70

PART - A

(Compulsory Question)

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1 Answer the following: (10 X 02 = 20 Marks)

(a) Write the axioms of probability.

(b) A fair die is rolled 5 times. Find the probability that "six" will show 2 times.

(c) State central limit theorem.

(d) Define correlation coefficient.

(e) A random process $X(t) = A\,Sin\omega_0 t$, where ω_0 is constant and 'A' is a uniform random variable over the interval (0, 1). Find whether X(t) is a stationary process or not.

(f) State autocorrelation properties.

(g) Find the PSD if $R_{XX}(\tau)$ is given as $e^{-2\lambda|\tau|}$.

(h) Calculate the noise equivalent bandwidth of the filter defined with transfer function: $H(f) = \frac{1}{1+J2\pi fRC}$.

(i) For a random variable with a CDF: $F_X(x) = (1 - e^{-x})\,u(x)$. Find $\Pr(X > 5)$ and $\Pr(X > 5/X < 7)$.

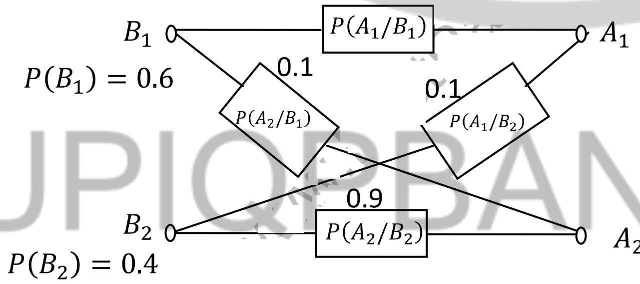
(j) State Wiener – Khintchine theorem.

PART - B

(Answer all five units, 5 X 10 = 50 Marks)

UNIT - I

2 (a) A binary symmetric channel is shown in below. Find the probability of (i) A_1, (ii) A_2, (iii) $P(B_1/A_1)$. (iv) $P(B_2/A_2)$. (v) $P(B_1/A_2)$. (vi) $P(B_2/A_1)$.



(b) List the properties of conditional density function.

OR

3 (a) Write and plot probability density function and probability distribution function of the following random variables:

(i) Uniform random variable.

(ii) Exponential random variable.

(iii) Laplace random variable.

(iv) Rayleigh random variable.

(b) A random variable X is defined as below, over the interval (0, 1). Find its conditional CDF of X given that $X < \frac{1}{2}$; $F_X(x) = \begin{cases} 0, & x < 0 \\ x, & 0 \le x < 1 \\ 1, & x > 1 \end{cases}$

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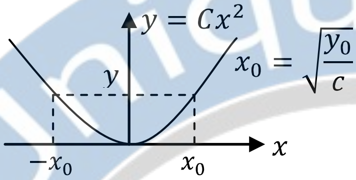
UNIT - II

4 (a) Find $f_Y(y)$ for the square law transfamation Y = T(X) = Cx^2 shown below.



(b) Find whether the two random variables X, and Y are statistically independent or not if the joint p.d.f is given by $f_{XY}(x,y) = \frac{1}{12}\, u(x)\, u(y)\, e^{-\left(\frac{x}{4}\right)-\left(\frac{y}{3}\right)}$.

OR

5 (a) Find the p.d.f of a random variable W defined as sum of X, Y with densities shown below;

$$f_X(x) = \frac{1}{a}[u(x) - u(x-a)]$$

$$f_Y(y) = \frac{1}{b}[u(y) - u(y-b)]$$

With a<b

(b) An exponential random variable has a p.d.f as shown below $f_X(x) = be^{-bx}\, u(x)$ with mean value $\frac{1}{b}$. Find its coefficient of skewness and kurtosis.

UNIT - III

6 (a) Two random process X(t) and Y(t) defined as below

$$X(t) = A\, Cos\omega_0 t + B\, Sin\omega_0 t$$

$$Y(t) = B\, Cos\omega_0 t - A\, Sin\omega_0 t$$

Where A, B are uncorrelated random variables with mean ‘0’ and same variance and ω_0 is constant. Find whether X(t) and Y(t) are jointly wide-sense stationary or not.

(b) A random process $X(t) = a\, Sin(\omega_0 t + \theta)$ where θ is uniform over $[0, 2\pi]$. Find whether it is ergodic or not.

OR

7 (a) Find the mean, variance of the process X(A), with ACF given as $R_{XX}(\tau) = 25 + \frac{4}{1+6\tau^2}$.

(b) Define Poisson random process and list the conditions. Write the p.d.f and find its mean and variance.

UNIT - IV

8 (a) State the properties of power density spectrum.

(b) Find power spectrum of WSS noise process N(t) with autocorrelation function defined as below. $R_{NN}(\tau) = Pe^{-3|\tau|}$

OR

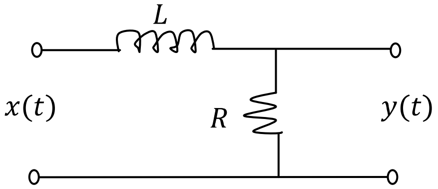
9 (a) List the properties of cross-power density spectrum.

(b) Find the cross-correlation function for a cross-power density spectrum given below:

$$f_{XY}(\omega) = \frac{8}{(\alpha + j\omega)^3}$$

UNIT - V

10 Find the output power for the LTI system shown below with input power spectral density $f_{XY}(\omega) = \frac{N_0}{2}$.



OR

11 For LTI system with impulse response h(t), input X(t), and output Y(t). Prove the following:

(i) $\mu_Y(t) = \mu_X\, H(0)$ (ii) $R_{YY}(\tau) = R_{XX}(\tau) * h(\tau) * h(-\tau)$

(iii) $f_{YY}(f) = f_{XX}(f)\; |H(f)|^2$ (iv) $f_{XY}(f) = f_{XX}(f)\, H(f)$

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