

MATHEMATICS – I
(Common to all branches)

Time: 3 hours

Max. Marks: 70

PART – A
(Compulsory Question)

- 1 Answer the following: (10 X 02 = 20 Marks)
- Solve $[\cos x \tan y + \cos(x+y)]dx + [\sin x \sec^2 y + \cos(x+y)]dy = 0$.
 - Solve $(x+1)\frac{dy}{dx} - y = e^x(x+1)^2$.
 - Find the particular integral of $(D^2 + 4D + 4)y = \frac{e^{-2x}}{x^2}$.
 - Solve $(x^2 D^2 - xD + 1)y = 0$.
 - Find the radius of curvature for the curve $y = 4\sin x - \sin 2x$ at $x = 90^\circ$.
 - If $u = \frac{y^2}{x}$, $v = \frac{x^2}{y}$ find $\frac{\partial(u,v)}{\partial(x,y)} = ?$
 - Evaluate $\int_0^a \int_0^{\sqrt{ay}} xy \, dx \, dy$.
 - Evaluate $\int_0^\pi \int_0^{a \cos \theta} r \sin \theta \, dr \, d\theta$.
 - Find the unit vector normal to the surface $x^2 - y^2 + z = 2$ at the point $(1, -1, 2)$.
 - Show that $\vec{F} = (y^2 + 2xz^2)\vec{i} + (2xy - z)\vec{j} + (2x^2z - y + 2z)\vec{k}$ is irrotational.

PART – B
(Answer all five units, 5 X 10 = 50 Marks)**UNIT – I**

- Solve $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = x^2 + 3$.
 - Prove that the system of confocal and coaxial parabolas $y^2 = 4a(x+a)$ is self orthogonal.
- Solve $(D^2 + 4D + 3)y = e^x \sin x$.
 - Solve $\frac{dy}{dx} + \frac{x}{1-x^2}y = x\sqrt{y}$.

UNIT – II

- Solve $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$ by using method of variation of parameters.

OR

- Solve $(x+2)^2 \frac{d^2y}{dx^2} - (x+2) \frac{dy}{dx} + y = 3x + 4$.

UNIT – III

- Expand $\sin(xy)$ in powers of $(x-1)$ and $(y-\pi/2)$ upto second degree terms.
 - Discuss the maximum and minimum of $f(x, y) = x^3 + y^3 - 12x - 3y + 20$.

OR

- If $x = r \sin \theta \cos \theta$, $y = r \sin \theta \sin \theta$ and $z = r \cos \theta$ find $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$.
 - Show that the rectangular solid of maximum volume that can be inscribed in a sphere is a cube.

UNIT – IV

- Evaluate $\iint_R (x^2 + y^2) dx dy$, where R is the square $0 \leq x \leq a$, $0 \leq y \leq a$.
 - Transform the integral into polar-co-ordinates and hence evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{x^2 + y^2} dy dx$.

OR

- Change the order of integration in $\int_0^a \int_x^a (x^2 + y^2) dy dx$ and then evaluate.
 - Find the area included between the curves $y^2 = 4x$ and $x^2 = 4y$.

UNIT – V

- If $\vec{F} = (3x^2 + 6y)\vec{i} - 14yz\vec{j} + 20xz^2\vec{k}$, evaluate $\int_C \vec{F} \cdot d\vec{r}$ where C is the straight line joining $(0,0,0)$ to $(1,1,1)$.
 - Using Stokes theorem, evaluate $\int_C \vec{F} \cdot d\vec{r}$ for the function $\vec{F} = x^2\vec{i} + xy\vec{j}$ in XOY-plane bounded by $x = 0$, $y = 0$, $x = a$, $y = a$.

OR

- Verify Divergence theorem for $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$ taken over the rectangular parallelepiped $x = 0$, $x = 1$, $y = 0$, $y = 2$, $z = 0$ and $z = 3$.
