Code: 15A54402 **R15**

B.Tech II Year II Semester (R15) Regular & Supplementary Examinations May/June 2018

MATHEMATICS – IV

(Common to EEE, ECE and EIE)

Time: 3 hours Max. Marks: 70

PART – A

(Compulsory Question)

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1 Answer the following: (10 X 02 = 20 Marks)

(a) Define Gamma and Beta functions.

(b) Evaluate $\Gamma\left(\frac{5}{2}\right)$ and $\Gamma(10)$.

(c) Prove the recurrence relations: $J_n^1(x) = \frac{1}{2}[J_{n-1}(x) - J_{n+1}(x)]$.

(d) Show that $P_2(\cos\theta) = \frac{1}{4}(1 + 3\cos 2\theta)$.

(e) Write Harmonic function in Cartesian and Polar form.

(f) Show that $W = e^z$ is analytic function.

(g) State generalized Cauchy's integral formula.

(h) Determine the poles and order for the function $\frac{z+4}{(z-1)^2(z-2)^3}$.

(i) State Cauchy Residue theorem.

(j) Write Laurent's series expansion.

PART – B

(Answer all five units, 5 X 10 = 50 Marks)

UNIT – I

2 (a) Show that $\int_0^\infty \sqrt{y}\, e^{-y^2}\, dy \times \int_0^\infty e^{-y^2}/\sqrt{y}\, dy = \frac{\pi}{2\sqrt{2}}$.

(b) Compute $\int_0^{\frac{\pi}{2}} \sqrt{\cot\theta}\, d\theta$ by expressing in terms of gamma functions.

OR

3 Solve $4x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$ by the method of series solution.

UNIT – II

4 Using suitable recurrence relation, obtain the expression for the functions:

(i) $J_{3/2}(x)$. (ii) $J_{-3/2}(x)$. (iii) $J_{5/2}(x)$. (iv) $J_{-5/2}(x)$.

OR

5 Derive Rodrigue's formula. Hence compute $P_0(x)$, $P_1(x)$, $P_2(x)$, $P_3(x)$ and $P_4(x)$.

UNIT – III

6 State and prove Cauchy- Riemann equation in polar form.

OR

7 Discuss the transformation $w = \sin z$.

UNIT – IV

8 Compute $\int_C (\bar{z})^2\, dz$ where C is the circle: (i) $|z| = 1$. (ii) $|z-1| = 1$.

OR

9 State Cauchy's theorem, using Cauchy's integral formula compute $\int_C \frac{\sin\pi z^2 + \cos\pi z^2}{(z-1)(z-2)}\, dz$ $C: |z| = 3$.

UNIT – V

10 Evaluate $\int_C \frac{z-1}{(z+1)^2(z-2)}\, dz$, where $C: |z-1| = 2$ using Cauchy's residue theorem.

OR

11 (a) Determine the poles and their residue at $\frac{\sin z}{(2z-\pi)^2}$.

(b) Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in Laurent series valid for $|z| < 1$.

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