

I B. Tech II Semester Supplementary Examinations, Nov/Dec - 2018
MATHEMATICS-III

(Com. to all branches)

Time: 3 hours

Max. Marks: 70

- Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B**)
 2. Answering the question in **Part-A** is Compulsory
 3. Answer any **THREE** Questions from **Part-B**

PART -A

1. a) Reduce to echelon form and hence find the rank of the matrix (4M)
- $$A = \begin{bmatrix} 1 & 2 & -4 & 5 \\ 2 & -1 & 3 & 6 \\ 8 & 1 & 9 & 7 \end{bmatrix}$$
- b) Prove that the sum of the given values of a matrix is the trace of A and product of the Eigen values of A is the determinant of A. (4M)
- c) Find the perimeter of the cardioid $r = a(1 - \cos \theta)$ (4M)
- d) Find $\Gamma\left(\frac{1}{2}\right)$ (3M)
- e) Find grad ϕ where $\phi(x, y, z) = \log(x^2 + y^2 + z^2)$ at (1,1,1) (3M)
- f) Find work done in moving particle in the force field $\vec{F} = 3x^2\vec{i} + (2xz - y)\vec{j} + z\vec{k}$ along the space curve $x = 2t^3, y = t, z = 4t^2 - t$ from $t = 0$ to $t = 1$. (4M)

PART -B

2. a) Express the following system in matrix form and solve by Gauss elimination method. (8M)
- $$\begin{aligned} 2x_1 + x_2 + 2x_3 + x_4 &= 6 ; \\ 6x_1 - 6x_2 + 6x_3 + 12x_4 &= 36 ; \\ 4x_1 + 3x_2 + 3x_3 - 3x_4 &= 1 ; \\ 2x_1 + 2x_2 - x_3 + x_4 &= 10. \end{aligned}$$
- b) Show that the system of equations : (8M)
- $$\begin{aligned} 2x_1 - 2x_2 + x_3 &= \lambda x_1; \\ 2x_1 - 3x_2 + 2x_3 &= \lambda x_2 ; \\ -x_1 + 2x_2 &= \lambda x_3 \end{aligned}$$
- can possess a non-trivial solution only if $\lambda = 1$ $\lambda = -3$ and obtain the general solution in each case.
3. a) Reduce the Q.F. $3x^2 - 2y^2 - z^2 - 4xy + 12yz + 8xz$ to the Canonical form (8M)
- b) Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ (8M)



4. a) Evaluate $\iint x^3 y \, dx \, dy$ over the region enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in the first quadrant. (8M)

b) Evaluate $\int_0^b \int_0^{\frac{a}{b}\sqrt{b^2-y^2}} xy \, dx \, dy$ by changing the order of the integration. (8M)

5. a) Show that (8M)

$$(i) \quad \Gamma(x)\Gamma(-x) = \frac{-\pi}{x \sin \pi x}$$

$$(ii) \quad \Gamma\left(\frac{1}{2} + x\right)\Gamma\left(\frac{1}{2} - x\right) = \pi \operatorname{Sec} \pi x$$

b) Show that $\beta(m, n) = \int_0^{\frac{\pi}{2}} \sin^{2m-1} \theta \cos^{2n-1} \theta \, d\theta$ and deduce that (8M)

$$\int_0^{\frac{\pi}{2}} \sin^n \theta \, d\theta = \int_0^{\frac{\pi}{2}} \cos^n \theta \, d\theta = \frac{\Gamma\left(\frac{n+1}{2}\right)\sqrt{\pi}}{2\Gamma\left(\frac{n+2}{2}\right)}$$

6. a) Find the values of a, b, c if the directional derivative of the function $\phi = ax^2 + byz + cz^2x^3$ at the (1,2,-1) has the maximum magnitude 64 in the direction parallel to z axis. (8M)

b) Prove that $\nabla^2 (\log r) = \frac{2}{r^2}$ (8M)

7. a) Verify stoke's theorem for $\vec{F} = x^2 \vec{i} + xy \vec{j}$ around the square in $z = 0$ plane whose sides are along the lines $x = 0; y = 0; x = 1, y = 1$. (8M)

b) Evaluate $\oint_c \sin y \, dx + x(1 + \cos y) \, dy$ by using green's theorem over the ellipse (8M)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1; z = 0.$$

