10. (a) Let D be a right ideal of R. Then show that D is dense if and only if $\underset{0\neq r_1\in R}{\forall}\ \underset{r_2\in R}{\forall}\ \underset{r\in R}{\exists}\ (r_1 r\neq 0$ and $r_2 r\in D)$.

(b) Let M_R be a finite dimensional. Then show that the ring H of endomorphisms of I_R is semiperfect.

M.Sc. DEGREE EXAMINATION, APRIL 2018.

Fourth Semester

Mathematics

Paper I — NON COMMUNICATIVE RINGS

(Regulation 2012)

Time : Three hours   Maximum : 70 marks

Answer ONE question from each Unit.

All questions carry equal marks.

UNIT I

1. (a) Show that a ring R is primitive if and only if there exists a faithful irreducible module A_R.

(b) Show that the prime radical of R is the set of all strongly nilpotent elements.

Or

2. (a) Let K and P be ideals such that $K\subset P\subset R$. Then show that P/K is prime if and only if P is prime.

(b) Show that every primitive ideal is a prime ideal.

UNIT II

3. (a) If $e^2 = e \in R$ and $f^2 = f \in R$ then show that $eR \cong fR$ if and only if there exits $u, v \in R$ such that $uv = e$ and $uv = f$.

   (b) Show that the radical of a right Artinian ring is nilpotent.

Or

4. (a) Show that a ring R is completely reducible and simple if and only if it is the ring of all linear transformations of a finite dimensional vector space.

   (b) Show that, if R is Artinian then $Rad\,R = rad\,R$.

UNIT III

5. (a) Let N be a nilideal of R. Then show that idempotents modulo N can be lifted.

   (b) If e is an idempotent of R and $N = Rad\,R$, then show that $Rad(e\text{Re}) = e\text{Re} \cap N = eNe$.

Or

6. (a) Show that any Artinian ring is semiperfect.

   (b) If R is semiperfect and e is a primitive idempotent of R, then show that $e\text{Re}$ is local.

UNIT IV

7. (a) If M is the direct sum of a family of modulus $\{M_i / i \in I\}$, then show that M is projective if and only if each M_i is projective.

   (b) Show that every module is isomorphic to a submodule of the character module of a free module.

Or

8. (a) Show that every R-module is projective if and only if R is completely reducible.

   (b) Show that an Abelian group is injective if and only if its is divisible.

UNIT V

9. Prove that the following conditions are equivalent :

   (a) ${}_H H \cong {}_H I$ canonically

   (b) $I_R \cong Q_R$ canonically

   (c) $H \cong Q$ canonically as rings

   (d) Q_R is injective

   (e) $I_Q \cong Q_Q$ canonically

   (f) Q_Q is injective.

Or

UNIT V

9. State and prove Caratheodory theorem.

Or

10. (a) Assume that $\{E_i\}$ is a sequence of disjoint measurable sets and $E = \cup E_i$. Then show that for any set A. $\mu^*(A \cap E) = \sum \mu^*(A \cap E_i)$.

(b) Let $\{(A_i \times B_i)\}$ be a countable disjoint collection of measurable rectangles whose union is a measurable rectangle $A \times B$. Then show that $\lambda(A \times B) = \sum \lambda(A_i \times B_i)$.

M.Sc. DEGREE EXAMINATION, APRIL 2018.

Fourth Semester

Mathematics

Paper II — MEASURES AND INTEGRATION

(Regulation 2012)

Time : Three hours Maximum : 70 marks

Answer ONE Questions from each Unit.

All questions carry equal marks.

UNIT I

1. (a) Show that if E_1 and E_2 are measurable, then $m(E_1 \cup E_2) + m(E_1 \cap E_2) = mE_1 + mE_2$.

(b) Prove that if f is a measurable function and $f = g$ $a.e$ then g is measurable.

Or

2. (a) Let $\{E_i\}$ be a sequence of measurable sets. Then prove that $m(\cup E_i) \leq \sum mE_i$.

If the sets E_n are pair wise disjoint, then $M(\cup E_i) = \sum mE_i$.

(b) Prove that if $m^* E = 0$ then E is measurable.

UNIT II

3. (a) Show that if f is integrable over E, then so is $|f|$ and $\left|\int_E f\right| \leq \int_E |f|$

(b) State and prove Monotone convergence theorem.

Or

4. (a) Let f be non-negative measurable function. Then show that $\int f = 0$ implies $f = 0$ a.e.

(b) Let ϕ and ψ be simple functions which vanish outside a set of finite measure. Then prove that $\int a\phi + b\psi = a\int \phi + b\int \psi$ and if $\phi \geq \psi$ a.e. then $\int \phi \geq \int \psi$.

UNIT III

5. (a) Prove that a function f is of bounded variation on $[a,b]$ if and only if f is the difference of two monotone real valued functions on $[a,b]$.

(b) Prove that a function f is absolutely continuous on $[a,b]$ and $f'(x) = 0$ a.e. then f is constant.

Or

6. Let f be an increasing real valued function on the interval $[a,b]$. Then prove that f is differentiable almost every where. Also the derivative f' is measurable and $\int_a^b f'(x)\,dx \leq f(b) - f(a)$.

UNIT IV

7. (a) State and prove Hahn decomposition theorem.

(b) Let $\{A_n\}$ be a countable collection of measurable sets. Then prove that $\mu\left(\bigcup_{k=1}^{\infty} A_k\right) = \lim_{n\to\infty} \mu\left(\bigcup_{k=1}^{h} A_k\right)$.

Or

8. (a) Let $(X, \mathcal{B})$ be a measurable space and $\{\mu_n\}$ be a sequence of measures on $\mathcal{B}$ such that for each $E \in \mathcal{B}$, $\mu_{n+1} E \geq \mu_n E$. Let $\mu E = Lim\, \mu_n E$. Then prove that μ is a measures on $\mathcal{B}$.

(b) State and prove Lebesgue convergence theorem.

(b) Let $T : X \to X$ be a compact linear operator on a normed X, and let $\lambda \neq 0$. Then prove that there exists an integer r such that $X = V(T_\lambda^r) \oplus T_\lambda^r(X)$.

Or

10. Let $T : X \to X$ be a compact linear operator on a normed linear space X, and let $\lambda \neq 0$. Then prove that the equations $T_x = \lambda_x$ and $T^* f - \lambda f = 0$ have the same ruler of linearly independent solutions.

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(MAT40312/16)

M.Sc. DEGREE EXAMINATION, APRIL 2018.

Fourth Semester

Mathematics

Paper III — OPERATORY THEORY

(Regulation 2012)

Time : Three hours Maximum : 70 marks

Answer ONE question from each Unit.

All questions carry equal marks.

UNIT I

1. (a) State and prove Schwartz equality.

(b) Show that an orthonormal set M in a Hilbert space H is total in H if and only if for all $x \in H$ the personal equation $\sum_K |< x, eK >|^2 = \|x\|^2$ holds.

Or

2. (a) Prove that if Y is a closed subspace of a Hilbert space H, then $Y = Y^{\perp\perp}$

(b) State and prove Bessel inequality.

UNIT II

3. (a) If P_n is the Legendre polynomial of order n. Then prove that $\|P_n\| = \sqrt{\frac{2}{2x+1}}$.

(b) Let $\{T_n\}$ be a sequence of bounded self-adjoint linear operators $T_n : H \to H$ on a Hilbert space H. Suppose that $\{T_n\}$ converges to T, that is $\|T_n - T\| \to 0$ where $\|.\|$ is the norm of the space $B(H,H)$. Then prove that the limit operator T is a bounded self-adjoint linear operator on H.

Or

4. State and prove Riesz's theorem of functionals on Hilbert spaces.

UNIT III

5. (a) Show that the resolvent set $\rho(T)$ of bounded linear operator T on a complex Banach space X is open.

(b) Let X be a Banach space, $T \in B(X,X)$. If $\|T\| < 1$ then prove that $(1-T)^{-1}$ exists a bounded linear operator on the whole space X and $(1-T)^{-1} = 1 + T + T^2 + ...$

Or

6. State and prove spectral mapping theorem for polynomials.

UNIT IV

7. (a) Define :

(i) A Banach algebra

(ii) Resolvent set

(iii) Spectrum

(iv) Spectral radius.

(b) Let A be a complex banach algebra with identity e. Then for any $x \in A$, prove that the spectrums $\sigma(x)$ is compact and the spectral radius satisfies $r_a(x) \leq \|x\|$.

Or

8. Let x and y be normed spaces and $T : x \to y$ be a linear operator. Then prove that T is compact if and only if it maps every bounded sequence $\{x_n\}$ in x onto a sequence $\{Tx_n\}$ in y which has a convergent.

UNIT V

9. (a) Let $T : X \to X$ be a compact linear operator on a normal space X and $\lambda \neq 0$. Show hat $T_x - \lambda_x = y$ has a solution x if and only if y is such that $f(y) = 0$ for all $f \in x'$ satisfying $T^X f - \lambda f = 0$.

(MAT404A12/16)

M.Sc. DEGREE EXAMINATION, APRIL 2018.

Fourth Semester

Mathematics

Paper IV — ALGEBRAIC CODING THEORY

(Regulation 2012)

Time : Three hours Maximum : 70 marks

Answer ONE question from each Unit.

All questions carry equal marks.

UNIT I

1. (a) Explain correcting and detecting error patterns.

(b) For any $u, v, w \in k^n$, prove the following.

(i) $wt(u+v) \leq wt(u) + wt(v)$

(ii) $d(u,v) \leq d(u,v) + d(v,w)$.

Or

2. (a) Explain the finding the most likely codeword transmitted.

(b) Form the IMLD table for the code $C = \{101, 111, 011\}$.

UNIT II

3. (a) Explain error-detecting and error-correcting code.

(b) Define the distance of a code. If C is a code with distance d. Prove the following (i) C detects all error patterns of weight less than or equal to d-1 (ii) there is atleast one error pattern of weight d which C will not detect.

Or

4. (a) Find a basis for C^{+}, where $C =< S >$, $S = \{1010, 0101, 1111\}$.

(b) Explain linear codes in detail.

UNIT III

5. (a) Explain generating matrices and encoding.

(b) Find the other five generator matrices for the code generated by $\hat{G} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Or

6. (a) Explain reliability of IMLD for linear codes.

(b) Let C be a linear code with parity check matrix $H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$.

UNIT IV

7. (a) State and prove the hamming bound theorem.

(b) Explain decoding the extended Golay code.

Or

8. (a) Construct the PCM for the hamming code of length 7.

(b) Discuss about the reed-mullar codes.

UNIT V

9. (a) Explain about the polynomials and words.

(b) With usual notation, find all words v of length n such that $\pi(v) = v$.

Or

10. (a) Discuss about the finding cyclic codes.

(b) Let C be a linear cyclic code of length $n = Q$ and generator of polynomial $g(x) = 1 + x^3 + x^6$. Find the generator polynomial of the dual cyclic code C^{+}.

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(MAT405B12/16)

M.Sc. DEGREE EXAMINATION, APRIL 2018.

Fourth Semester

Mathematics

Paper V — OPERATIONS RESEARCH

(Regulation 2012)

Time : Three hours   Maximum : 70 marks

Answer ONE question from each Unit.

All questions carry equal marks.

UNIT I

1. Use dual simplex method to solve the following LLP :

Maximize : $Z = 5x_1 + 12x_2 + 4x_3$

Subject to : $x_1 + 2x_2 + x_3 \leq 5$

$2x_1 - x_2 + 3x_3 = 2$

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$

Or

2. (a) State the general rules for converting any primal LPP into its dual.

(b) Use dual simplex method to solve the L.P.P :

Max : $Z = -2x_1 - 2x_2 - 4x_3$

Subject to : $2x_1 + 3x_2 + 5x_3 \geq 2$

$3x_1 + x_2 + 7x_3 \leq 3$ and

$x_1, x_2, x_3 \geq 0.$

UNIT II

3. Solve the following by revised simplex method :

Maximize : $Z = x_1 + x_2 + 3x_3$

Subject to : $3x_1 + 2x_2 + x_3 \leq 3$

$2x_1 + x_2 + 2x_3 \leq 2$

$x_1, x_2, x_3 \geq 0.$

Or

4. Solve the following LPP by revised simplex method :

Maximize : $Z = 5x_1 + 3x_2$

Subject to : $4x_1 + 5x_2 \geq 10$

$5x_1 + 2x_2 \leq 10$

$3x_1 + 8x_2 \leq 12$ and

$x_1, x_2 \geq 0.$

UNIT III

5. (a) State the basic assumptions made in the theory of games. Explain the maximum criterion of optimality.

(b) Solve the following by using Dominance method.

Player B

		I	II	III	IV
Player A	1	18	4	6	4
	2	6	2	13	7
	3	11	5	17	3
	4	7	6	12	2

Or

6. (a) Discuss the importance of integer programming problem in optimization theory. Can an integer programming problem be solved by rounding-off the corresponding simplex solution?

(b) Find an optimum all integer solution to the following LPP :

Maximize : $Z = x_1 + 4x_2$

Subject to : $2x_1 + 4x_2 \leq 7$

$5x_1 + 3x_2 \leq 15$ and

$x_1, x_2 \geq 0$ are integers.

UNIT IV

7. There are five jobs to be assigned 5 machines and associated cost matrix as follows :

Machines

	1	2	3	4	5
A	11	17	8	16	20
B	9	7	12	6	15
C	13	16	15	12	16
D	21	24	17	28	26
E	14	10	12	11	15

Find the optimum assignment and associated cost using the assignment techniques.

Or

8. Find the optimal sequence of the following :

		Machine			
		M_1	M_2	M_3	M_4
Job	J_1	25	15	14	24
	J_2	22	12	20	22
	J_3	23	13	16	25
	J_4	26	10	13	29

UNIT V

9. Solve the following LP problem by using Dynamic programming approach :

Maximize : $Z = 3x_1 + 4x_2$

Subject to constraints : $2x_1 + 4x_2 \geq 40$

$2x_1 + 5x_2 \geq 180$ and

$x_1, x_2 \geq 0.$

Or

10. Explain about forward and backward recursion in dynamic programming.