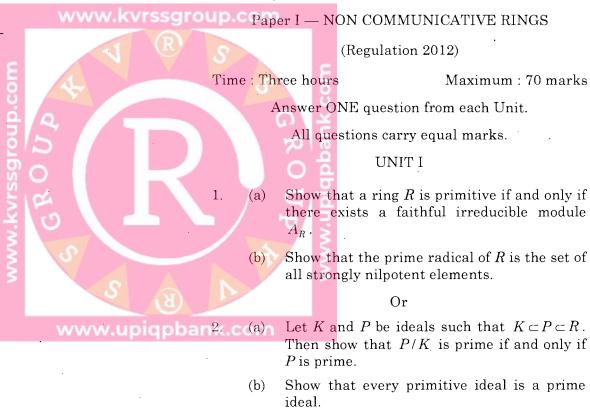
- 10. (a) Let D be a right ideal of R. Then show that D is dense if and only if $\forall \forall \exists (r_1r \neq 0)$ and $r_2r \in D$.
 - (b) Let M_R be a finite dimensional. Then show that the ring H of endomorphisms of I_R is semiperfect.

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- 3. (a) If $e^2 = e \in R$ and $f^2 = f \in R$ then show that $eR \cong fR$ if and only if there exits $u, v \in R$ such that uv = e and uv = f.
 - (b) Show that the radical of a right Artinian ring is nilpotent.

Or

- 4. (a) Show that a ring R is completely reducible and simple if and only if it is the ring of all linear transformations of a finite dimensional vector space.
 - (b) Show that, if R is Artinian then Rad R = rad R.

UNIT III

- 5. (a) Let N be a nilideal of R. Then show that idempotents modulo N can be lifted.
 - (b) If e is an idempotent of R and N = RadR, then show that $Rad(eRe) = eRe \cap N = eNe$.

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6.

Show that any Artinian ring is semiperfect.

(b) If R is semiperfect and e is a primitive idempotent of R, then show that eRe is local.

UNIT IV

- 7. (a) If M is the direct sum of a family of modulus $\{M_i/i \in I\}$, then show that M is projective if and only if each M_i is projective.
 - (b) Show that every module is isomorphic to a submodule of the character module of a free module.

Or

- 8. (a) Show that every *R*-module is projective if and only if *R* is completely reducible.
 - (b) Show that an Abelian group is injective if and only if its is divisible.

UNIT V

- Prove that the following conditions are equivalent:
 - (a) $_{H}H \cong_{H} I$ canonically
 - (b) $I_R \cong Q_R$ canonically
 - (c) $H \cong Q$ canonically as rings
- www.upiqpbank.co $^{(d)}$ Q_R is injective
 - (e) $I_Q \cong Q_Q$ canonically
 - (f) Q_Q is injective.

Or

3 (MAT40112/16)

9. State and prove Caratheodory theorem.

Or

- 10. (a) Assume that $\{E_i\}$ is a sequence of disjoint measurable sets and $E = \bigcup E_i$. Then show that for any set A. $\mu^*(A \cap E) = \sum \mu^*(A \cap E_i)$.
 - (b) Let $\{(A_i \times B_i)\}$ be a countable disjoint collection of measurable rectangles whose union is a measurable rectangle $A \times B$. Then show that $\lambda(A \times B) = \sum \lambda(A_i \times B_i)$.

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Paper II — MEASURES AND INTEGRATION (Regulation 2012)

Time: Three hours

Maximum: 70 marks

Answer ONE Questions from each Unit.

All questions carry equal marks.

UNIT I

- 1. (a) Show that if E_1 and E_2 are measurable, then $m(E_1 \cup E_2) + m(E_1 \cap E_2) = mE_1 mE_2$.
 - Prove that if f is a measurable function and $f \in g$ a.e then g is measurable.

Or

Let $\{E_i\}$ be a sequence of measurable sets. Then prove that $m | \cup E_i | \le \sum_i m E_i$.

If the sets E_n are pair wise disjoint, then $M(\bigcup E_i) = \sum mE_i$.

(b) Prove that if $m^*E = 0$ then E is measurable.

(MAT40212/16)

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- Show that if f is integrable over E, then so is 3. |f| and $|\int_{a}^{b} f| \leq \int_{a}^{b} |f|$
 - (b) State and prove Monotone convergence theorem.

Or

- Let f be non-negative measurable function. 4. Then show that $\int f = 0$ implies f = 0 a.e.
 - Let ϕ and ψ be simple functions which vanish outside a set of finite measure. Then that $\int a\phi + b\psi = a\int \phi + b\int \psi$ and if $\phi \ge \psi$ a.e. then $\phi \ge \psi$.

UNIT III-

5. Prove that a function f is of bounded variation on [a,b] if and only if f is the difference of two monotone real valued functions on [a,b].

Prove that a function f is absolutely continuous on [a,b] and f'(x) = 0 a.e. then f is constant.

Or

6.

8.

Let f be an increasing real valued function on the interval [a,b]. Then prove that f is differentiable almost every where. Also the derivative f' is measurable and $\int_{a}^{b} f'(x) dx \le f(b) - f(a)$.

UNIT IV

- State and prove Hahn decomposition (a) theorem.
 - Let A be a countable collection of measurable Then sets. prove that $\mu\left(\bigcup_{n\to\infty}^{\infty}A_{n}\right)=\lim_{n\to\infty}\mu\left(\bigcup_{n\to\infty}^{\infty}A_{n}\right).$

Or

- Let (X, \mathcal{B}) be a measurable space and $\{\mu_n\}$ be a sequence of measures on b such that www.upiqpbank.con each $E \in \mathfrak{B}$ $\mu_{n-1} E \ge \mu_n E$ Let $\mu E = Lim \mu_n E$ Then prove that μ is a measures on **b**.
 - and prove Lebesgue convergence State theorem.

3

Let $T: X \to X$ be a compact linear operator on a normed X, and let $\lambda \neq 0$. Then prove that there exists an integer r such that $X = V(T_1^r) \oplus T_1^r(X)$.

Or

(MAT40312/16)

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Mathematics

Let $T: X \to X$ be a compact linear operator on arssgroup com Paper III — OPERATORY THEORY normed linear space X, and let $\lambda \neq 0$. Then prove that the equations $T_r = \lambda_r$ and $T^*f - \lambda f = 0$ have the same ruler of linearly independent solutions.

4

(Regulation 2012)

Time: Three hours

Maximum: 70 marks

Answer ONE question from each Unit.

All questions carry equal marks.

UNIT I

- State and prove Schwartz equality. (a)
- Show that an orthonormal set M in a Hilbert (b) space H is total in H if and only if for all $x \in H$ the personal equation $\sum_{K} \left| \langle x, eK \rangle \right|^2 = \left\| x \right\|^2 \text{ holds.}$

Or

- 2. Prove that if Y is a closed subspace of a Hilbert space H, then $Y = Y^{\perp \perp}$
 - State and prove Bessel inequality.

(MAT40312/16)

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- If P_n is the Legendre polynomial of order n. 3. Then prove that $||P_n|| = \sqrt{\frac{2}{2r+1}}$.
 - Let $\{T_n\}$ be a sequence of bounded selfadjoint linear operators $T_n: H \to H$ on a Hilbert space H. Suppose that $\{T_n\}$ converges to T, that is $||T_n - T|| \to 0$ where ||.|| is the norm of the space B(H,H). Then prove that the limit operator T is a bounded self-adjoint linear operator on H.

Or

State and prove Riesz's theorem of functionals on 4. Hilbert spaces.

UNIT III

- 5. Show that the resolvent set $\rho(T)$ of bounded linear operator T on a complex Banach space X is open.
 - (b) Let X be a Banach space, $T \in B(X,X)$. If ||T|| < 1 then prove that $(1-T)^{-1}$ exists a bounded linear operator on the whole space X and $(1-T)^{-1} = 1 + T + T^2 + ...$

Or

6. State and prove spectral mapping theorem for polynomials.

UNIT IV

- 7. Define: (a)
 - A Banach algebra
 - Resolvent set (ii)
 - (iii) Spectrum
- group.com_(iv) Spectral radius.
 - Let A be a complex banach algebra with identity e. Then for any $x \in A$, prove that the spectrums $\sigma(x)$ is compact and the spectral radius satisfies $r_a(x) \leq ||x||$.

Or

8. Let x and y be normed spaces and $T: x \to y$ be a linear operator. Then prove that T is compact if and only if it maps every bounded sequence $\{x_n\}$ in x onto a sequence $\{Tx_n\}$ in y which has a convergent.

UNIT V

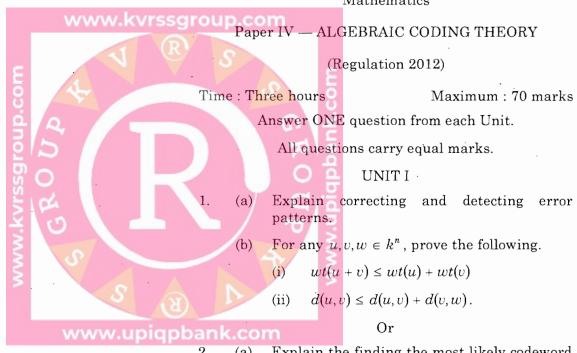
Let $T: X \to X$ be a compact linear operator on a normal space X and $\lambda \neq 0$. Show hat $T_x - \lambda_x = y$ has a solution x if and only if y is such that f(y) = 0 for all $f \in x'$ satisfying $T^X f - \lambda f = 0.$

(MAT404A12/16)

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Fourth Semester

Mathematics



- 2. (a) Explain the finding the most likely codeword transmitted.
 - (b) Form the IMLD table for the code $C = \{101,111,011\}$.

- 3. (a) Explain error-detecting and error-correcting code.
 - (b) Define the distance of a code. If C is a code with distance d. Prove the following (i) C detects all error patterns of weight less than or equal to d-1 (ii) there is at least one error pattern of weight d which C will not detect.

Or

- 4. (a) Find a basis for C^+ , where $C = \langle S \rangle$, $S = \{1010, 0101, 1111\}$.
 - (b) Explain linear codes in detail.

UNIT III

- 5. (a) Explain generating matrices and encoding.
 - (b) Find the other five generator matrices for the code generated by $\vec{G} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Or

- 6. (a) Explain reliability of IMLD for linear codes.
 - Let C be a linear code with parity check matrix $H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$.

2 (MAT404A12/16)

UNIT IV

- 7. (a) State and prove the hamming bound theorem.
 - (b) Explain decoding the extended Golay code.

Or

- 3. (a) Construct the PCM for the hamming code of length 7.
 - (b) Discuss about the reed-mullar codes.

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10.

UNIT V

- (a) Explain about the polynomials and words.
- (b) With usual notation, find all words v of length n such that $\pi(v) = v$.

Or

(a) Discuss about the finding cyclic codes.

3

Let C be a linear cyclic code of length n = Q and generator of polynomial $g(x) = 1 + x^3 + x^6$. Find the generator polynomial of the dual cyclic code C^+ .

(MAT404A12/16)

(MAT405B12/16)

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Fourth Semester

Mathematics

Paper V — OPERATIONS RESEARCH

(Regulation 2012)

Time: Three hours

Maximum: 70 marks

Answer ONE question from each Unit.

All questions carry equal marks.

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1. Use dual simplex method to solve the following LLP:

Maximize: $Z = 5x_1 + 12x_2 + 4x_3$

Subject to : $x_1 + 2x_2 + x_3 \le 5$

$$2x_1 - x_2 + 3x_3 = 2$$

$$x_1 \ge 0, x_2 \ge 0, x_3 \ge 0.$$

Or

- 2. (a) State the general rules for converting any primal LPP into its dual.
 - (b) Use dual simplex method to solve the L.P.P:

 $Max: Z = -2x_1 - 2x_2 - 4x_3$

Subject to : $2x_1 + 3x_2 + 5x_3 \ge 2$

$$3x_1 + x_2 + 7x_3 \le 3$$
 and

$$x_1, x_2, x_3 \ge 0.$$

UNIT II

3. Solve the following by revised simplex method:

Maximize: $Z = x_1 + x_2 + 3x_3$

Subject to : $3x_1 + 2x_2 + x_3 \le 3$

$$2x_1 + x_2 + 2x_3 \le 2$$

 $x_1, x_2, x_3 \ge 0.$

4. Solve the following LPP by revised simplex method:

Maximize:
$$Z = 5x_1 + 3x_2$$

Subject to :
$$4x_1 + 5x_2 \ge 10$$

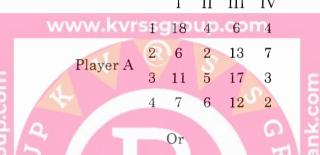
$$5x_1 + 2x_2 \le 10$$

$$3x_1 + 8x_2 \le 12 \text{ and}$$

$$x_1,x_2 \geq 0.$$

UNIT III

- State the basic assumptions made in the theory of games. Explain the maximum 5. (a) criterion of optimality.
 - Solve the following by using Dominance method. (b)



- Discuss the importance of integer programming problem in optimization theory. Can an 6. (a) integer programming problem be solved by rounding-off the corresponding simplex solution?
 - (b) Find an optimum all integer solution to the following LPP:

$$Maximize: Z = x_1 + 4x_2$$

Subject to :
$$2x_1 + 4x_2 \le 7$$

$$5x_1 + 3x_2 \le 15$$
 and

$$x_1, x_2 \ge 0$$
 are integers.

UNIT IV

7. There are five jobs to be assigned 5 machines and associated cost matrix as follows:

Machines

Find the optimum assignment and associated cost using the assignment techniques.

8. Find the optimal sequence of the following:

$$M_1$$
 M_2 M_3 M_4

$$J_1$$
 25 15 14 24

$$J_{
m ob}$$
 J_2 22 12 20 22

$$J_3$$
 23 13 16 25

$$J_4$$
 26 10 13 29

UNIT V

9. Solve the following LP problem by using Dynamic programming approach:

Maximize :
$$Z = 3x_1 + 4x_2$$

$$2x_1 + 4x_2 \ge 40$$

Subject to constraints: $2x_1 + 5x_2 \ge 180$ and SSGroup.com

$$x_1, x_2 \ge 0.$$

Or

10. Explain about forward and backward recursion in dynamic programming.

