(i) Find the extinction probability for this branching process.

(ii) Give a necessary and sufficient conditions for sure extinction.

Or

8. (a) Derive probability generating functions of branching processes.

(b) For a G.W. branching process, show that $P_n(S) = P_{n-1}(P(S))$ and $P_n(S) = P(P_{n-1}(S))$.

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M.Sc. DEGREE EXAMINATION, APRIL 2019.

Second Semester

Statistics

Paper I — STOCHASTIC PROCESSES

(Regulation 2015)

Time : Three hours Maximum : 70 marks

Answer ONE question from each Unit.

(18 marks each for Unit–I and Unit–II, 17 marks each for Unit–III, IV)

UNIT I

1. (a) Classify the stochastic process by space and time domain.

(b) What are the fundamental properties of a first-state, first-order Markov chains?

Or

2. (a) Verify the Chapman-Kolmogorov equation from the following transition matrix of a Markov chain for m = n = 1 :

$$P = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \end{bmatrix}.$$

(b) What are countable-state markov chains? Explain its properties.

UNIT II

3. (a) What is poisson cluster process? Explain in detail.

(b) Explain the properties of poisson process. Explain the postulates of poisson process.

Or

4. (a) Give an example of a positive recurrent birth-death rates. Also write its process.

(b) For a poisson process (with parameter λ), show that $P(N(t) = n) = e^{-\lambda t}(\lambda t)^n / n!$, $n = 0,1,2,....$

UNIT III

5. (a) State and prove Wald's equation.

(b) If $g : [0, \infty] \to [0, \infty)$ is such that :

(i) $g(t) \geq 0$ for all t,

(ii) $\int_0^a g(t)dt < \infty$

(iii) g is a non-increasing function, then prove that

$$\int_0^t g(t - x)dm(x) \to \frac{1}{\mu}\int_0^\infty g(x)dx \quad \text{as}$$

$t \to \infty$.

Or

6. (a) State and prove central limit theorem for renewals.

(b) Show that the time Sr upto n^{th} event in a renewal process is continuous time is asymptotically normally distributed.

UNIT IV

7. (a) What is branching? Explain its applications in brief.

(b) In a branching process, the distribution is given by its characteristic function $P(S) = as^r + bs + c$, when $a, b, c > 0$ and $a + b + c = 1$.

8. (a) Write the application of a sequential test.

(b) Let x be $N(\theta, 0)$ and is the notation of this $\theta' = 4$, $\theta'' = 9$, $\alpha_{\alpha} = 0.05$, $\beta_{\alpha} = 0.10$. Show that the sequential ratio test can be based upon the statistic $\sum_1^4 X_j^{\iota}$ also determine $C_0(4)$ and $C_1(4)$.

M.Sc. DEGREE EXAMINATION, APRIL 2019.

Second Semester

Statistics

Paper II — TESTING OF HYPOTHESIS

(Regulation 2015)

Time : Three hours Maximum : 70 marks

Answer ONE question from each Unit.

(18 marks each for Unit-I and Unit-II, 17 marks each for Unit-III and Unit-IV)

UNIT I

1. (a) What is a power function? Explain in detail.

(b) Consider a normal distribution of the form $N(\theta, 4)$. The simple hypothesis $H_0 : \theta = 0$ is rejected and the alternate composite hypothesis $H_1 : \theta > 0$ is accepted if and only if the observed mean $\overline{X}$ of a random sample of size 25 is $\geq \frac{3}{5}$. Find the power function $\gamma(\theta)$, $0 \leq \theta$.

2. (a) State and prove Neyman-Pearson lemma.

(b) What is type I and type II errors is testing of hypothesis? Explain with examples.

UNIT II

3. (a) Let $X_1, X_2, \ldots X_n$ be a random sample from a Bernoulli $b(1, \theta)$ distribution, where $0 < \theta < 1$. Show that the likelihood ratio test of $H_0 : \theta = \theta_0$ versus $H_1 : \theta \neq \theta_0$ is based upon the statistic $Y = \sum_{i=1}^{n} X_i$ also obtain the null distribution of y.

(b) Explain the properties of Likelyhood ratio test.

4. (a) Write the necessary and sufficient conditions for the existence of a similar region. Indicate the relationship between similar region and unbiased critical region.

(b) Explain UMPU tests for testing various parameters of a normal distribution with unit variance.

UNIT III

5. (a) Explain the difference between χ^{v} – test and Kolmogorov and Smirnov test.

(b) Explain the Bartlett's test for homogeneity of variance.

6. (a) A random sample of size $n = 27$ from a bivarate normal distribution yielded a sample correlation co-efficient of $r = -0.45$. Would the hypothesis $H_0 : \rho - 0$ be rejected in favour of $H_1 : \rho \neq 0$ at an $\alpha = 0.05$?

(b) What is the confidence interval of ρ for a random variable of size 12 from a bivariate normal distribution yielded a correlations co-efficient of $r = 0.6$.

UNIT IV

7. (a) Write the properties of wald's fundamental identify.

(b) Determine α, β curves for testing Poisson proportions.

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Second Semester

Statistics

Paper III — THEORY OF LINEAR ESTIMATION AND ANALYSIS OF VARIANCE

(Regulation 2015)

Time : Three hours | Maximum : 70 marks

Answer ONE question from each Unit.

(18 marks each for Unit I and Unit II, 17 marks for each Unit III and Unit IV)

UNIT I

1. (a) Explain Rank of a matrix, Inverse of a matrix with examples.

   (b) Explain how a positive definite matrix can be reduced into diagonal form using a non-singular matrix.

2. (a) State and prove Cauley-Hamilton theorem.

   (b) Show the Matrix $M = \begin{bmatrix} 1 & 3 & 5 \\ 0 & 3 & 2 \\ 2 & 6 & 12 \end{bmatrix}$ has an Inverse.

UNIT II

3. (a) What is a Linear Estimation? Explain how various parameters are calculated with linear Estimation.

   (b) Explain Gauss-Markov set up in detail.

4. (a) If $E_Y = X_B$ and $Cov(Y) = \sigma^2 I$, then prove that the least squares Estimators $\hat{B}_j, J = 1, 2, \ldots K$ have minimum variance among all linear unbiased Estimators. Prove it by Gauss-Markov theorem,

   (b) State and prove a necessary and sufficient condition for the estimability of a linear parametric function.

UNIT III

5. (a) Write the Mathematical model of one-way classification of ANOVA with Equal number of observations.

   (b) Explain Fisher's least significance difference test with an example.

6. (a) What is a Multiple comparision? Explain the concept with two examples.

   (b) Explain Fixed, random, mixed models. Also write their properties.

UNIT IV

7. (a) Write the mathematical structure of Analysis of covariance for Two-way classification.

   (b) Explain the analysis of Co-variance of LSD with one missing observation.

8. (a) Explain the analysis of Covariance of RBD with one missing value.

   (b) Explain the analysis of Covariance of CRD with one con-comitant observation.

(STA20415)

M.Sc. DEGREE EXAMINATION, APRIL 2019.

Second Semester

Statistics

Paper IV — MULTIVARIATE ANALYSIS

(Regulation 2015)

Time : Three hours Maximum : 70 marks

Answer ONE question from each Unit.

(18 marks each for Unit–I and Unit–II, 17 marks each for Unit–III, IV)

UNIT I

1. (a) Define multivariate analysis. What are the objectives of multivariate analysis and explain its importance?

(b) What is a p-variate normal distribution? and write its characteristics.

Or

2. (a) Consider a population with probability function given by $f(x/\theta) = \theta(1-\theta)^x$, $(x = 0, 1, 2, 3, \ldots)$ with $0 < \theta < 1$. It is known that $E(X) = (1-\theta)/\theta$ a random of size 1000 gave $\sum_{i=1}^{1000} x_i = 980$. Derive maximum likelihood estimator of θ.

(b) Derive the distribution of a linear combination of a p-variate normal random variable.

UNIT II

3. (a) Write the properties of invariance of T^r-statistics.

(b) Derive Wishart's distribution and write it's properties.

Or

4. (a) Write the mathematical model to test the difference of two vectors.

(b) Explain the difference between ANOVA and MANOVA. Also write the applications of MANOVA.

UNIT III

5. (a) Compare and contrast discriminant analysis and cluster analysis.

(b) Write the procedure to classify and discrimation between two normal populations.

Or

6. (a) Write the procedure of Fisher's method for discrimination among several populations.

(b) Write a short on principle component analysis. Explain the use of factor loadings in the context of principle component analysis.

UNIT IV

7. (a) Give a brief description of how a cluster analysis might be used in practical situations.

(b) Explain various non-hierarchical clustering method.

Or

8. (a) Discuss the method of estimating canonical correlation and variates.

(b) Explain maximum likelihood method of estimation in deriving factor loadings.

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