

R16

Code No: 133BQ

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech II Year I Semester Examinations, November/December - 2018

SIGNALS AND STOCHASTIC PROCESS

(Common to ECE, ETM)

Time: 3 Hours

Max. Marks: 75

Note: This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A.

Part B consists of 5 Units. Answer any one full question from each unit.

Each question carries 10 marks and may have a, b, c as sub questions.

PART A

(25 Marks)

- 1.a) Is the system described by the equation $y(t) = x(2t)$ time invariant or not? Why? [2]
- b) Give the relation between bandwidth and Rise time of a signal. [3]
- c) What are the effects of aliasing and how can you minimize the aliasing error? [2]
- d) Distinguish between series and transform in the Fourier Representation of a signal. [3]
- e) Let $x(s) = \mathcal{L}\{x(t)\}$, determine the initial value, $x(0)$ and the final value $x(\infty)$, for the following signal using initial value and final value theorems. [2]

$$x(s) = \frac{7s+6}{s(3s+5)}$$

- f) How the stability of a system can be found in Z-Transform and what is the condition for causality in terms of Z-Transform. [3]
- g) Prove that $R_{xy}(\tau) = R_{yx}(-\tau)$. [2]
- h) If the customers arrive at a bank according to a Poisson process with mean rate 2 per minute, find the probability that during a 1-minute interval no customer arrives. [3]
- i) Prove that the power spectral density of a real random process is an even function. [2]
- j) Find the auto correlation function, whose spectral density is: [3]

$$S(\omega) = \begin{cases} \pi, & |\omega| \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

PART B

(50 Marks)

- 2.a) Prove that the set $\sin m\omega_0 t$ and $\sin n\omega_0 t$ are orthogonal for $m \neq n$, where $m = 0, 1, 2, \dots, \infty$ and $n = 0, 1, 2, \dots, \infty$, over to, $t_0 + \frac{2\pi}{\omega_0}$.

- b) Explain the concepts of unit step function and Signum function. [5+5]

OR

- 3.a) Explain causality and physical reliability of a system and explain Paley-wiener criterion.
- b) Consider a stable LTI system characterized by the differential equation:
 $\frac{dy(t)}{dt} + 2y(t) = x(t)$. Find its impulse response. [5+5]

4.a) Find the Fourier Transform of the signal $x(t) = e^at u(-2t)$.

b) Define sampling theorem for time limited signal and find the Nyquist rate for the following signals.

i) $\text{rect } 300t$ ii) $10 \cos 300\pi t$

[4+6]

OR

5.a) Derive the expression for trigonometric Fourier series coefficients.

b) Determine the exponential form of the Fourier series representation of the signal shown in figure 1.

[4+6]

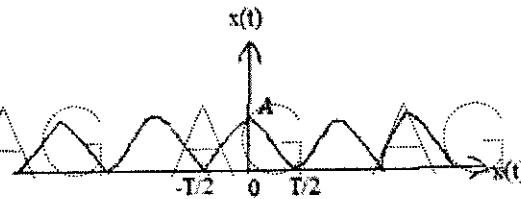


Figure 1

6.a) By using the power series expression technique, find the inverse Z-Transform of the following $X(z)$.

$$X(z) = \frac{z}{2z^2 - 3z + 1}; |z| < \frac{1}{2}$$

b) Distinguish between the Laplace, Fourier and Z-Transforms.

[7+3]

OR

7.a) Find the Laplace Transform of the periodic, rectangular wave shown in figure 2.

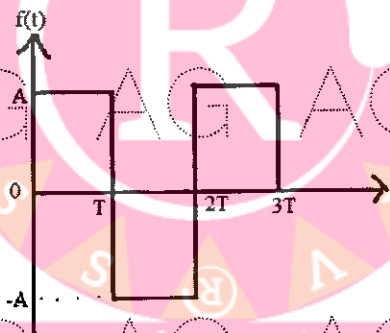


Figure 2

b) Find the Laplace Transform of following functions:

i) Exponential function

ii) Unit step function.

[6+4]

8.a) Explain the characteristics of a first order and strict sense stationary process using relevant expressions.

b) State and prove the properties of auto correlation of a random process. [5+5]

OR

9.a) Find the mean, variance and Root Mean Square value of the process, whose auto correlation function is $R_x(\tau) = \frac{25\tau^2 + 36}{6.25\tau^2 + 4}$.

b) Consider two random processes $x(t) = 3\cos(\omega t + \theta)$ and $y(t) = 2\cos(\omega t + \phi)$, where $\phi = \theta - \frac{\pi}{2}$ and θ is uniformly distributed over $(0, 2\pi)$, verify $|R_{xy}(\tau)| \leq \sqrt{R_{xx}(0)R_{yy}(0)}$.

[5+5]

10.a) Derive the relation between input and output power spectral densities of a linear system.

b) The cross power spectrum of real random process $x(t)$ and $y(t)$ is given by:

$$S_{xy}(\omega) = \begin{cases} a + ib\omega, & \text{if } |\omega| \leq 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find the cross correlation function.

[5+5]

OR

11.a) Consider a random process $X(t) = A_0 \cos(\omega_0 t + \theta)$, where A_0 and ω_0 are constants and θ is a uniform random variable in the interval $(0, \pi)$, find whether $X(t)$ is WSS process.

b) Show that $S_{yy}(\omega) = |H(\omega)|^2 S_{xx}(\omega)$. Where $S_{xx}(\omega)$ and $S_{yy}(\omega)$ are the power spectral density functions of the input $x(t)$ and the output $y(t)$ respectively and $H(\omega)$ is the system transfer function.

[5+5]