35052(OR)

M.Sc., DEGREE EXAMINATION, NOVEMBER 2016

THIRD SEMESTER

MATHEMATICS

Paper – II DISCRETE MATHEMATICS

(No additional sheet will be supplied)

Time: 3 hours Max. Marks: 90

PART-A (5 x 6 = 30 marks)

Answer any FIVE questions

Each question carries Three (3) marks

Each answer should not exceed One (1) page

1. Define a modular lattice. Show that every distributive lattice is modular but not conversely.
2. Obtain disjunctive normal forms of 7(PVQ) ⇆ (P∧Q)
3. Write about Lexicographic ordering
4. Let (S, *), (T, Δ) and (V, ⊕) be semigroups and g:s → T and L:T → V be semigroup homomorphism. Then show that hog:S → V is a semi group homomorphism from (S, *) to (V, ⊕)
5. Let (L, *, ⊕) be a distributive lattice show that for any a,b,c ϵ L (a*b = a*c) Λ (a⊕ b= a⊕c) ⇒ b=c
6. Define a sub-lattice. Show that every interval of a lattice is a sub-lattice.
7. In any Boolean algebra, prove that $(a + b^{1})\,(b + c^{1})\,(c + a^{1}) = (a^{1} + b)\,(b^{1} + c)\,(c^{1} + a)$
8. Obtain the value of the Boolean forms $x_1 * (x'_1 \oplus x_2)$ $x_1 * x_2$ and $x_1 \oplus (x_1 \oplus x_2)$

PART-B (4 x 15 = 60 marks)

Answer ALL questions

Each question carries Fifteen (15) marks

Each answer should not exceed Six (6) pages

9. a) Obtain the principal disjunctive normal form of P → (P→Q) ∧7 (7QV7P)
   b) Obtain the principal conductive normal form of (7P→R) ∧ (Q←P)

(OR)

10. a) Verify whether ((P→R) ∧ (Q→R)) → ((P∨Q) → R is a tautology.
    b) Show that R ∧ (P∨Q) is a valid conclusion from the premises P∨Q, Q→R, P→M and 7 M

[P.T.O]

11. a) Show that there are only five distinct Hasse diagrams for partially ordered sets that contains three elements.

b) Let R = {(1,2), (3,4), (2,2)} and S = {(4,2), (2,5), (3,1), (1,3)}. Find RoS, SoR, Ro (SoR), (RoS) OR, RoR, SoS and RoRoR.

(OR)

12. a) Show that the order of a subgroup of a finite group divide the order of the group.

b) Let (M, *) be a monoid. Prove that there exists a subset $T \subseteq M^M$ such that (M,*) is isomorphic to the monoid (T,o).

13. a) Define the direct product of two lattices and show that the different product of two lattices is itself a lattice.

b) Let $L_1 = \{1,2,4\}$, $L_2 = \{1,3,9\}$ denote the chains of divisors of 4 and 9 with the partial order relation 'division'. Derive the direct product Lattice if L_1 and L_2 and draw a diagram for it.

(OR)

14. a) Find all sublattices of the lattice (S_{12},D).

b) Show that the demorgan's laws, given by $(a*b)^1 = a^1 \oplus b^1$ and $(a \oplus b)^1 = a^1 * b^1$ hold in a complemented distribute lattice.

15. a) Use the Karnaugh map representation to find a minimal sum- of products expression of the following functions.

(i) $f(a,b,c) = \Sigma(0,1,4,6)$

(ii) $f(a,b,c,d) = \Sigma(0,1,2,3,13,15)$.

b) In any Boolean algebra, show that

$(a+b)(a^1+c) = a\,c + a^1 b = a\,c + a^1\, b + b\,c$

(OR)

16. a) Show that the symmetric functions form a Boolean algebra.

b) Obtain the sum-of-products, products-of-sums canonical forms of $x_1 x^1_2 + x_3$.