(35052)

M.Sc. DEGREE EXAMINATION, OCTOBER/NOVEMBER 2018.

THIRD SEMESTER

Applied Mathematics

Paper II — DISCRETE MATHEMATICS

Time : Three hours

Maximum : 75 marks

(No additional sheet will be supplied)

PART A — (5 × 3 = 15 marks)

Answer any FIVE questions.

Each question carries 3 marks.

1. Write in symbolic form the statement?
The Gop will be destroyed if there is a flood?

2. Write an equivalent formula for $(p \wedge (Q \rightleftarrows R) \vee (R \rightleftarrows P)$ which does not contain the biconditional.

3. Let $X = \{1, 2, 3, 4\}$ and $R = \{\langle x, y\rangle / x > y\}$. Draw the graph of R and also give its matrix.

4. Let $X = \{a, b, c, d, e\}$ and let $C = \{\{a, b\}, \{c\}, \{d\}, \{e\}\}$. Show that the partition C defines an equivalence relation on X.

5. Let $(L, *, \oplus)$ be a distributive lattice. For any $a, b, c \in L$ then show that
$$(a * b) = (a * c) \wedge (a \oplus b = a \oplus c) \Rightarrow b = c.$$

6. Define :
   (a) Order preserving relative.
   (b) Order-isomorphic.

7. Expand the following function into their canonical sum of products form $f_1(x, y, z) = xy + y\bar{z}$.

8. In an Boolean algebra, show that $a = 0 \Leftrightarrow ab' + a'b = 0$.

PART B — (4 × 15 = 60 marks)

Answer ALL questions.

Each question carries 15 marks.

9. (a) Construct the truth table for $(P \to Q) \wedge (Q \to P)$.

   (b) Show that $P \to (Q \to R) \Leftrightarrow P \to (\neg Q \vee R) \Leftrightarrow (P \wedge Q) \to R$.

Or

10. (a) Obtain disjunctive normal forms of

(i) $P \wedge (P \rightarrow Q)$;

(ii) $\neg(P \vee Q) \rightleftarrows (P \wedge Q)$

(b) Obtain principal conjunctive normal form of the formula S given by $(\neg P \rightarrow R) \wedge (Q \rightleftarrows P)$.

11. (a) Let R and S be two relations on a set of positive integers $I : R = \{\langle x, 2x\rangle / x \in I\}$ $S = \{\langle x, 7x\rangle / x \in I\}$. Find $R \circ S$, $R \circ R$, $R \circ R \circ R$ and $R \circ S \circ R$.

(b) Given the relation matrices MR and MS, find M_{ROS}, $M_{\tilde{R}}, M_{\tilde{S}}$, $M_{\tilde{ROS}}$ and show that $M_{\tilde{ROS}} = M_{\tilde{S}O\tilde{R}}$.

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \quad M_S = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}.$$

Or

12. (a) Let A be the set of factors of a particular positive integer m and let $\leq$ be the relation divides i.e., $\leq = \{< x, y > | x \in A \wedge y \in A \wedge (x \text{ divides } y)\}$. Draw Hasse diagrams for

(i) $m = 2$; (ii) $m = 6$; (iii) $m = 30$; (iv) $m = 210$; (v) $m = 12$; and (vi) $m = 45$.

(b) Let $(M,*)$ be a monoid. Then show that there exists a subset $T \subseteq MM$ such that $< M,* >$ is isomorphic to the monoid $< T, \circ >$.

13. (a) Let $\langle L, \leq \rangle$ be a lattice in which $*$ and $\oplus$ denote the operations of meet and join respectively. For any $a, b \in L$ then show that $a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$.

(b) Let $\langle L, \leq \rangle$ be a lattice. For any $a, b, c \in L$ the following property then show that $a \leq c \Leftrightarrow a \oplus (b * c) \leq (a \oplus b) * c$.

Or

14. (a) Show that every chain is a distributive lattice.

(b) Show that a lattice is distributive iff $(a * b) \oplus (b * c) \oplus (c * a) = (a \oplus b) * (b \oplus c) * (c \oplus a)$.

15. (a) Obtain the product of sums canonical forms of the Boolean expression is (i) $x_1 * x_2$; (ii) $x_1 \oplus x_2$; (iii) $(x_1 \oplus x_2)' * x_3$.

(b) Obtain the values of the boolean forms $x_1 * (x_1' \oplus x_2) x_1 * x_2$ and $x_1 \oplus (x_1 * x_2)$ over the ordered pairs of the two-element boolean algebra.

Or

16. Show that the following boolean expressions are equivalent to one another. Obtain their sum of products canonical form (a) $(x \oplus y) * (x' \oplus z) * (y \oplus z)$ (b) $(x * z) \oplus (x' + y) \oplus (y * z)$ (c) $(x \oplus y) * (x' \oplus z)$ (d) $(x * z) \oplus (x' * y')$.
