35052

M.Sc. DEGREE EXAMINATION, NOVEMBER 2016.

Third Semester

Mathematics

Paper II — DISCRETE MATHEMATICS

Time : Three hours | Maximum : 75 marks

(No additional sheet will be supplied)

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE of the following.

Each question carries 5 marks.

Each answer should not exceed 1 page.

1. Show that $S \vee R$ is tautologically implied by

   $(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S)$.

2. Translate into symbolic form the statement :

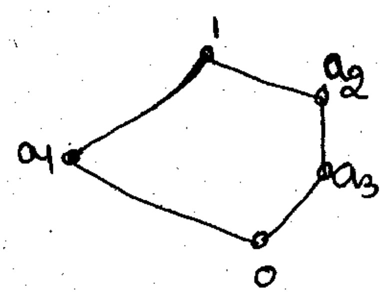
   Jack and Jill went up the hill.

3. Show that there are only five distinct phase diagrams for partially ordered sets that contains three elements.

4. Let $(S,*)$, (T,Δ) and $(V,\oplus)$ be semigroups and $g : S \rightarrow T$ and $L : T \rightarrow V$ be semigroup homormophism. Then show that $h \circ g : S \rightarrow V$ is a semigroup homomorphism from $(S,*)$ to $(V,\oplus)$.

5. Define a lattice, a distributive lattice and a complemented lattice. Show that every chain is distributive lattice.

6. Examine whether the lattice given by the diagram here under is distributive.



7. In any Boolean algebra, show that $a = b \Leftrightarrow ab' + a'b = 0$.

8. Determine the following function is symmetric $abc' + ab'c + a'bc + ab'c' + a'bc' + a'b'c$.

PART B — (4 × 12 ½ = 50 marks)

Answer ALL questions.

Each question carries 12½ marks.

Each answer should not exceed 5 pages.

UNIT I

9. (a) Prove that $(\exists x)(P(x) \wedge Q(x)) \Rightarrow (\exists x) P(x) \wedge (\exists x) Q(x)$.

(b) Show that : $(x)(P(x) \vee Q(x)) \Rightarrow (x) P(x) \vee (\exists x) Q(x)$.

Or

10. (a) Demonstrate that R is a valid inference from the premises $P \rightarrow Q, Q \rightarrow R$ and P.

(b) Show that $R \rightarrow S$ can be derived from the premises $P \rightarrow (Q \rightarrow S)$, $\urcorner R \vee P$ and Q.

UNIT II

11. (a) Let $(M, *)$ be a monoid. Prove that there exists a subset $T \subseteq M^M$ such that $(M, *)$ is isomorphic to the monoid (T, o).

(b) For any communicative monoid $(M, *)$, Show that the set of independent elemetns of M forms a submonoid.

Or

12. (a) If R is a partial ordering relation on a set X and $A \subseteq X$, show that $R \cap (A \times A)$ is partial ordering relation on A.

(b) Let $(S,*)$ be a given semigroup. Then show that there exists a homomorphism $g; S \rightarrow S^s$, where $(S^2,0)$ is a semigroup of functions from S to S under the operation of (left) composition

UNIT III

13. (a) Let $(L, \leq)$ be a lattice in which $*$ and $\oplus$ denote the operations of meet and join respectively. Then show that $a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$, for all $a, b \in L$.

(b) Let $(L, \leq)$ be a lattice. Then for any $a.b, c \in L$, show that the following inequalities hold

$a \oplus (b * c) \leq (a \oplus b) * (a \oplus c)$

$a * (b \oplus c) \geq (a * b) \oplus (a * c)$

Or

14. (a) Define a sub – lattice. Show that every interval of a lattice is a sub-lattice.

(b) Show that the Demorgan's Laws, given by $(a * b)' = a' \oplus b'$ and $(a \oplus b)' = a' * b'$ hold in a complements, distributive lattice.

UNIT IV

15. (a) In any Boolean algebra show that $(a + b)(a' + c) = ac + a'b = ac + a'b + bc$.

(b) Show that the Boolean expression $(x'_1 + x'_2) \oplus (x_1 \oplus x_2)$ is symmetric.

Or

16. Write the following Boolean expressions in an equivalent sum-of-products canonical form and product-of-sums canonical form.

(a) $x_1 * x_2$ (b) $x_1 \oplus x_2$ (c) $(x_1 \oplus x_2)' * x_2$.