35051

M.Sc. DEGREE EXAMINATION, NOVEMBER 2016.

THIRD SEMESTER

Mathematics

Paper I — FUNCTIONAL ANALYSIS

Time : Three hours — Maximum : 75 marks

(No additional sheet will be supplied)

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

Each question carries 5 marks.

Each answer should not exceed 1 page.

1. Define normed linear space. Prove that norm is continuous function.

2. State and prove Minkowski's inequality.

3. If T is an operator and T^* is its conjugate then prove that $\|T^*\| = \|T\|$.

4. Prove that a non-empty subset X of a normed linear space N is bounded iff $F(X)$ is a bounded set of numbers for each f in N^*.

5. Prove that a closed convex subject C of H contains a unique vector of smallest norm.

6. State and prove Bessel's inequality.

7. If N is normal operator on H then prove that $\|N^2\| = \|N\|^2$.

8. Prove that an operator T on H is unitary iff it is an isometric isomorphism of H onto itself.

PART B — ($4 \times 12\frac{1}{2} = 50$ marks)

Answer ALL questions.

Each question carries $12\frac{1}{2}$ marks.

Each answer should not exceed 5 pages.

9. If M is a closed linear subspace of a normed linear space N. The quotient space N/M defined by $\|x + M\| = \inf\{\|x + m\| : m \in M\}$ then prove that N/m is normed linear space. Further, if N is a banach space, Then so is N/M.

Or

10. State and prove Hahn – banach theorem.

11. State and prove Open mapping theorem.

Or

12. State and prove that Banach – Steinhaus theorem.

13. State and prove Riesz representation theorem.

Or

14. (a) Explain the Gram – Schmidt orthogonalization process.

(b) If M is a closed linear subspace of H then prove that $H = M \oplus M^{\perp}$.

15. (a) Prove that an operator T on H is self-adjoint iff $\langle Tx, x \rangle$ is real for all x.

(b) Prove that an operator T on H is normal iff $\|T^* x\| = \|T x\|$ for each $x \in H$.

Or

16. (a) Prove that a closed linear subspace M of H is invariant under an operator T iff $M^{\perp}$ is invariant under T^*.

(b) If T is normal then prove that the M_i's are pair wise orthogonal.