45052 (OR)

M.Sc. DEGREE EXAMINATION, APRIL 2018

Applied Mathematics

FOURTH SEMESTER

Paper II — GRAPH THEORY

Time : Three hours Maximum : 90 marks

(No additional sheet will be supplied)

PART A — (5 × 6 = 30 marks)

Answer any FIVE questions.

Each question carries 6 marks.

Each answer should not exceed 1 page.

1. Can a simple undirected graph of nine vertices have 39 edges? Explain

2. Show that number of vertices of odd degree in a graph is always even.

3. Define Tree and spanning Tree of a graph and give examples of each. What is difference between them.

4. How many possible spanning trees can a Complete Graph with n vertices have? Explain with an example.

5. Explain Travelling Salesman Problem by using Graph Theory.

6. Show that a finite connected graph is Eulerian if and only if each vertex has even degree.

7. State and prove Euler's Formula for planar graphs.

8. Define Hamiltonian Path and Hamiltonian Cycle and explain them with examples.

PART B — (4 × 15 = 60 marks)

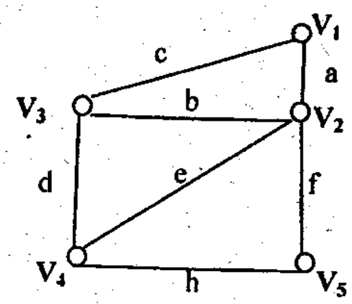
Answer ALL questions.

Each question carries 15 marks

Each answer should not exceed 6 pages.

9. (a) Define Bipartite Graph and Complete Bipartite Graph and give examples of each. State when can a Bipartite Graph become a Complete Bipartite Graph.

(b) Define sub graph and draw sub graphs of the open walks and closed walks of the following, if any



Or

10. (a) If M is the adjacency matrix for the graph G, then show that the $(i,j)^{th}$ entry of the matrix M^2 is contains the number of paths of length 2 which connect vertex Ito vertex j in G.

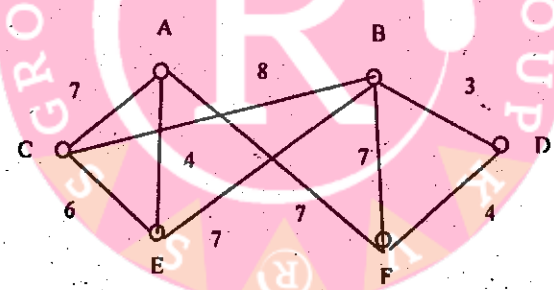
(b) Write the fusion algorithm for connectedness.

11. (a) Prove that a graph G with n vertices, $n-1$ edges and no circuit is connected.

(b) Prove that a Cut-set and any spanning tree must have at least one edge in common.

Or

12. Find the shortest spanning tree of the following graph by using Kruskal's Algorithm.



13. (a) Can there be a Hamiltonian graph which is also Eulerian? If so give all example.

(b) A graph G has a Hamiltonian circuit if $m \geq (n^2 - 3n + 6)/2$, *where* n and in denote the number of vertices and edges of G respectively.

Or

14. Explain Chinese Postman Problem with example.

15. Statement: If $G = (V., E)$ is a connected simple planar graph with n vertices and e edges,

Where $n \geq 3$, then $e \leq 3n - 6$

Prove or disprove the above statement.

Is the converse of this statement true or false? Explain with an example.

Or

16. The graph G is planar if and only if it does not contain a Kuratowski subgraph.