45051 (OR)

M.Sc. DEGREE EXAMINATION, APRIL 2018.

Applied Mathematics

FOURTH SEMESTER

Paper I — LABESGUE MEASURE AND INTEGRATION

Time : Three hours

Maximum : 90 marks

(No additional sheet will be supplied)

PART A — ($5 \times 6 = 30$ marks)

Answer any FIVE of the following.

Each question carries 6 marks.

Each answer should not exceed 1 page.

1. If E_1 and E_2 are measurable sets, then show that $E_1 \cup E_2$ is measurable.

2. Let f be a function with measurable domain D. Show that f is measurable iff the function of defined by $g(x) = f(x)$ for $x \in D$ and $g(x) = 0$ for $x \notin D$ is measurable.

3. Let f be a bounded function defined on $\{a, b\}$. If f is Riemann integrable on $[a, b]$ then show that it is measurable and $R\int_a^b f(x)\,dx = \int_a^b f(x)\,dx$. With the usual notation.

4. Prove that a bounded function f on $[a, b]$ is Riemann integrable if and only if the set of points at which f is discontinuous has measure zero.

5. Show that if f is bounded variation on $[a, b]$ the $T_a^b = P_a^b + N_a^b$.

6. Show that the sum and difference of two absolutely continuous functions are also absolutely continuous.

7. State and prove the Minkowski's inequality.

8. Prove that every convergent sequences is a cauchy sequence.

PART B — ($4 \times 15 = 60$ marks)

Answer ALL questions.

Each question carries 15 marks.

Each answer should not exceed 6 pages.

9. (a) Prove that the outer measure of an interval is its length.

(b) Show that if E_1 and E_2 are measurable, then

$$m(E_1 \cup E_2) + m(E_1 \cap E_2) = m(E_1) + m(E_2).$$

Or

10. (a) State and prove Monotone convergence theorem.

(b) State and prove Lebesgue convergence theorem.

11. (a) Let f be a bounded function defined on $\{a,b\}$. If f is Riemann integrable on $[a, b]$, then show that it is measurable and $R\int_a^b f(x)dx = \int_a^b f(x)dx$.

(b) Show that if $|f|$ is integrable over E, then so is $|f|$ and $\left|\int_E f\right| \le \int_E |f|$. Does the integrability of $|f|$ imply that of f? Justify your claim.

Or

12. (a) Prove that a normed linear space X is completed if and only if every absolutely summable series is summable.

(b) Let f be a bounded linear functional on L^ρ, $1 \le \rho < \infty$. Then prove that there exists a function g in L^1 such that $F(f)\int fg$ and $\|F\| = \|g\|_1$.

13. (a) Prove that a function f is of bounded variation on $[a, b]$ if and only if f is the differences of two monotone real valued functions on $[a, b]$.

(b) Let f be integrable on $[a, b]$. Then show that the function f defined by $F(X) = \int_a^X f(t)dt$ is a continuous function of bounded variable on $[a, b]$.

Or

14. (a) Let f be an integrable function on $[a, b]$ and suppose that $F(X) = F(a) + \int_a^X f(t)dt$. Then show that $F'(X) = F(X)$ for almost all X in $[a, b]$.

(b) If f is absolutely continuous, then show that f has a derivative almost everywhere.

15. (a) State and prove Holder's inequality.

(b) Show that L^ρ is complete $(1 \le \rho < \infty)$.

Or

16. State prove Riesz representation theorem.