

45051

M.Sc. DEGREE EXAMINATION, APRIL 2018.

Applied Mathematics

FOURTH SEMESTER

Paper I — LABESGUE MEASURE AND INTEGRATION

Time : Three hours

Maximum : 75 marks

(No additional sheet will be supplied)

PART A — (5 × 5 = 25 marks)

Answer any FIVE of the following.

Each question carries 5 marks.

Each answer should not exceed 1 page.

1. If E_1 and E_2 are measurable sets, then show that $E_1 \cup E_2$ is measurable.
2. Let f be a function with measurable domain D . Show that f is measurable iff the function of defined by $g(x) = f(x)$ for $x \in D$ and $g(x) = 0$ for $x \notin D$ is measurable.
3. Let f be a bounded function defined on $[a, b]$. If f is Riemann integrable on $[a, b]$ then show that it is measurable and $R \int_a^b f(x) dx = \int_a^b f(x) dx$. With the usual notation.
4. Prove that a bounded function f on $[a, b]$ is Riemann integrable if and only if the set of points at which f is discontinuous has measure zero.
5. Show that if f is bounded variation on $[a, b]$ the $T_a^b = P_a^b + N_a^b$.
6. Show that the sum and difference of two absolutely continuous functions are also absolutely continuous.
7. State and prove the Minkowski's inequality.
8. Prove that every convergent sequences is a cauchy sequence.

PART B — (4 × 12½ = 50 marks)

Answer ALL questions.

Each question carries 12½ marks.

Each answer should not exceed 6 pages.

9. (a) Prove that the outer measure of an interval is its length.
(b) Show that if E_1 and E_2 are measurable, then
$$m(E_1 \cup E_2) + m(E_1 \cap E_2) = m(E_1) + m(E_2).$$

Or

10. (a) State and prove Monotone convergence theorem.
 (b) State and prove Lebesgue convergence theorem.
11. (a) Let f be a bounded function defined on $\{a, b\}$. If f is Riemann integrable on $[a, b]$, then show that it is measurable and $R \int_a^b f(x) dx = \int_a^b f(x) dx$.
- (b) Show that if $|f|$ is integrable over E , then so is f and $\left| \int_E f \right| \leq \int_E |f|$. Does the integrability of $|f|$ imply that of f ? Justify your claim.

Or

12. (a) Prove that a normed linear space X is completed if and only if every absolutely summable series is summable.
- (b) Let f be a bounded linear functional on L^p , $1 \leq p < \infty$. Then prove that there exists a function g in L^1 such that $F(f) \int fg$ and $\|F\| = \|g\|_1$.
13. (a) Prove that a function f is of bounded variation on $[a, b]$ if and only if f is the differences of two monotone real valued functions on $[a, b]$.
- (b) Let f be integrable on $[a, b]$. Then show that the function f defined by $F(X) = \int_a^X f(t) dt$ is a continuous function of bounded variable on $[a, b]$.

Or

14. (a) Let f be an integrable function on $[a, b]$ and suppose that $F(X) = F(a) + \int_a^X f(t) dt$. Then show that $F'(X) = f(X)$ for almost all X in $[a, b]$.
- (b) If f is absolutely continuous, then show that f has a derivative almost everywhere.
15. (a) State and prove Holder's inequality.
 (b) Show that L^p is complete ($1 \leq p < \infty$).

Or

16. State prove Riesz representation theorem.