

(No additional sheet will be supplied)

## PART A — (5 × 3 = 15 marks)

Answer any FIVE questions.

Each question carries 3 marks.

Each answer should not exceed 1 page.

1. Define beta and gamma functions.
2. Show that
  - (a)  $J_0'(x) = -J_1(x)$  and
  - (b)  $2J_0''(x) = J_2(x) - J_0(x)$ .
3. Write the linearity property of Fourier transforms.
4. Find the Laplace transform of  $F(t)$ , where  $F(t) = \begin{cases} \cos\left(t - \frac{2\pi}{3}\right), & t > \frac{2\pi}{3} \\ 0 & t < \frac{2\pi}{3} \end{cases}$
5. What are symmetric and anti-symmetric tensors?
6. What is the method of separation of variables?
7. Mention the Cauchy-Riemann equations.
8. Expand  $f(z) = \frac{z}{(z+1)(z+2)}$  in a Taylor's series about  $Z=0$ . Determine the region of convergence.

PART B — (4 × 15 = 60 marks)

Answer ALL questions.

Each question carries 15 marks.

Each answer should not exceed 6 pages.

9. (a) Obtain the Rodrigue's formula for Hermite polynomial.

(b) Prove that  $\int_0^{\infty} x^m e^{-x} L_n(x) dx = (-1)^n n! \delta_{mn}$ .

Or

10. (a) Derive the equation for Bessel function.

(b) Prove the Legendre recurrence relation  $P_n^1(x) - xP_{n-1}^1(x) = nP_{n-1}(x)$ .

11. (a) State and prove the Laplace transform of integral of a function.

(b) Evaluate  $L^{-1}\left\{\frac{S}{(S^2+1)^3}\right\}$ .

Or

12. (a) Explain the Fourier transform of a delta function.

(b) Find the solution for the triangular wave using Fourier series.

13. (a) Distinguish between contra variant and covariant tensors.

(b) Give the transformation laws of tensors of rank 3.

Or

14. (a) Explain the method of separation of variables.

(b) State and prove the quotient law of tensors.

15. (a) State and prove Cauchy's integral theorem.

(b) State and prove Cauchy's integral theorem. Evaluate  $\oint_c \frac{dz}{z}$ .

Or

16. (a) State and prove Residue theorem.

(b) Obtain the residue of  $\frac{Z^4}{(Z-1)^4(Z-2)(Z-3)}$  at  $Z=1$ .