

(No additional sheet will be supplied)

PART A — (5 × 3 = 15 marks)

Answer any FIVE questions.

Each question carries 3 marks.

Each answer should not exceed 1 page.

1. Prove that $\Gamma(x+1) = x!$.
2. Find the generating function for the Legendre Polynomial.
3. What are the properties of Fourier transforms?
4. Find the Laplace transform of $2 \sin 2t \cos 4t$.
5. Explain the method of separation of variables in a long cylinder.
6. Define the contravariant, covariant and mixed tensors.
7. State and Prove Cauchy's theorem.
8. What is analytic function? Determine whether $f(z) = \frac{1}{z}$ is analytic or not.

PART B — (4 × 15 = 60 marks)

Answer ALL questions.

Each question carries 15 marks.

Each answer should not exceed 6 pages.

9. Write the Legendre's differential equation and find its solution.

Or

10. (a) Prove that $\int_0^{\frac{\pi}{2}} \cos^{2x-1} \theta \sin^{2y-1} \theta d\theta = \frac{\Gamma(x)\Gamma(y)}{2\Gamma(x+y)}$.
- (b) Define Beta and Gamma functions. Show that $\beta(x, y) = \beta(y, x)$.

11. (a) State and prove the modulation theorem.
(b) Find the solution for (i) Unit step function (ii) Impulse function.

Or

12. (a) What are the properties of Laplace transforms?
(b) Find the Laplace transform of $(1 + \sin 2t)$.
13. What is the wave equation? Derive the equation for the vibrations of a rectangular membrane.

Or

14. (a) What is dummy suffix notation?
(b) Define Symmetric and anti symmetric tensors.
(c) Discuss how the tensor theory is applied to thermal expansion.
15. (a) State and prove Cauchy – Riemann equations.
(b) Show that the function defined by $f(z) = \sqrt{|xy|}$ satisfies the Cauchy – Riemann equation at the origin but is not analytic at that point.

Or

16. (a) State and prove Cauchy Integral formula.
(b) Evaluate $\int_C \frac{e^z}{(z-1)(z-4)} dz$.