45054

M.Sc. DEGREE EXAMINATION, APRIL 2018.

Applied Mathematics

FOURTH SEMESTER

Paper IV — MATHEMATICAL STATISTICS

Time : Three hours

Maximum : 75 marks

(No additional sheet will be supplied)

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

Each question carries 5 marks.

Each answer should not exceed 1 page.

1. Define independent events and exhaustive events and give example of each.
2. A bag contains 30 balls numbered from 1 to 30. One ball is drawn at random. Find the probability that the number of the drawn ball will be a multiple of (a) 5 or 9 (b) 5 or 6..
3. Define Probability Density Function and Probability Mass function.
4. Find the Moment Generating Function for the Poisson Distribution.
5. Find the mean and variance of Bernoulli random variable X using its generating Function.
6. Find the probability that a random variable having the standard normal distribution will take on a value (a) between 0.87 and 1.28 (b) greater than 0.85.
7. Explain consistent estimate and efficient estimate.
8. Explain Multinomial Distribution.

PART B — (4 × 12 ½ = 50 marks)

Answer ALL questions.

Each question carries 12 ½ marks.

Each answer should not exceed 6 pages.

9. (a) A manufacturing firm produces units of a product in 4 plants. Define event A_i a unit is produced in plant i=1,2,3,4 and event B: a unit is defective. From the past records of the proportions of defectives produced at each plant the following conditional probabilities are set: $P\left(\frac{B}{A_1}\right)=0.05$, $P\left(\frac{B}{A_2}\right)=0.1$, $P\left(\frac{B}{A_3}\right)=0.15$, $P(B/A_4)=0.02$. The first plant produces 30% of the units of the product, the second plant 25%, third plant 40% and fourth plant 5%. A unit of the product made at one of these plants is tested and is found to be defective. What is the probability that the unit was produced in (i) plant I (ii) plant 3.

(b) If the probability density of a random variable is given by

$$f(x) = \begin{cases} x, & \text{for } 0 < x < 1 \\ 2 - x, & \text{for } 1 \le x < 2, \\ 0, & \text{elsewhere} \end{cases}$$, find the probability that a random variable will take on a value (i) between 0.2 and 0.8 (ii) between 0.6 and 1 .2.

Or

10. (a) State and prove addition Law of Probability.

(b) Define Joint Density function, Joint Distribution Function, Marginal Density Function and Marginal Distribution Function. If the joint density function of a two dimension random variable is

$$f(x, y) = \begin{cases} 6e^{-2x-3y}, & \text{for } x > 0, y > 0 \\ 0, & \text{elsewhere} \end{cases}$$

Find the marginal densities of the random variables x and y. Also determine whether two random variables are independent.

11. (a) Find the mean and variance of Binomial Distribution. Also find the conditions for Binomial Distribution tend to Normal Distribution.

(b) The number of typing mistakes made by a secretary has a Poisson distribution. The mistakes are made independently at an average rate of 1.65 per page. Find the probability that a three-page letter contains no mistakes.

Or

12. (a) The finish times for marathon runners during a race are normally distributed with a mean of 195 minutes and a standard deviation of 25 minutes. (i) What is the probability that a runner will complete the marathon within 3 hours? (ii) What proportion of the runners will complete the marathon between 3 hours and 4 hours?

(b) The probability that a driver must stop at any one traffic light coming to Lincoln University is 0.2. There are 15 sets of traffic lights on the journey. (i) What is the probability that a student must stop at exactly 2 of the 15 sets of traffic lights? (ii) What is the probability that a student will be stopped at 1 or more of the 15 sets of traffic lights?

13. State and Prove Central Limit Theorem.

Or

14. Suppose $X_1, X_2, \ldots X_n$ are independent random variables such that X_1 is $N(\mu_1, \sigma_1^2)$. X_2 is $N(\mu_2, \sigma_2^2), \ldots\ldots X_n$ is $N(\mu_n, \sigma_n^2)$. Then find whether the sum of the random variables $Y = X_1 + X_2 + \ldots + X_n$ is also normally distributed? If so find the mean and variance.

Also discuss the case when $X_1, X_2, \ldots X_n$ are not independent random variables.

15. Explain χ^2 test. Also write the limitations of χ^2 test.

Or

16. State and Prove Neyman-Pearson Lemma.