15151

M.Sc DEGREE EXAMINATION DECEMBER 2015
I SEMESTER
Statistics
PAPER I: PROBABILITY AND DISTRIBUTION

Time : 3hours (No additional sheet will be supplied) **Max Marks : 75**

PART -A (5x3=15 marks)
Answer any FIVE questions.
Each question carries THREE (3) marks.
Each answer should not exceed ONE (1) page.

1. Define (i) Classes of sets (ii) Field.
2. Define probability measure and give its properties
3. Explain central limit theorem for I.I.D random variables along with its applications.
4. State Lindberg-levy central limit theorem and discuss its application.
5. With usual notation, show that F is unique if $F_n \xrightarrow{w} F$.
6. Define moment generating function and its properties.
7. Define't' distribution and its properties.
8. What are the properties of partial correlation coefficient?

PART -B (4x15=60 marks)
Answer all questions.
Each question carries FIFTEEN (15) marks.
Each answer should not exceed SIX (6) pages

9. (a) Define σ –field. Show that a σ –field is a monotone field and conversely?
   (b)If f and g are measurable functions, prove that Max {f, g} and Min {f, g} are also measurable.

(OR)

10. (a) Define (i) Measure and (ii) Measurable function. Show that the intersection of an arbitrary number of σ –fields is a σ –field.
    (b) Explain (i) Lebesque (ii) Lebesque-Stieltjes (iii) Borel σ –field in R^k and (iv) Induced σ –field with suitable example.

11. (a) Define (i) Random variable. (ii) Distribution Function (iii) Characteristic function. State and prove simple properties of characteristic function.
    (b) State Lindeberg-Fellar condition. Let $x_1,x_2,\ldots\ldots$ are independent random variables. Such that there exists a constant A with $P\{|X_n| \leq A\}=1$ for all 'n'. Obtain the conditions under which Lindeberg condition is satisfied.

(OR)

12. (a) State and prove Liapounor from of central limit theorem?
    (b) Let $x_1,x_2,\ldots\ldots$ are independent random variables and suppose $p\{X_k=a_k\}= p\{X_k=-a_k\}=1/2, a_k=k^{\lambda}$ $(k=1,2,\ldots)$. Deduce the conditions for the Central limit theorem to hold.

[P.T.O]

13. (a) Define (i) Mathematical expertation ii) conditional expertation. State and prove the moment problem.

(b) State and prove the necessary and sufficient condition of W.L.L.N.

(OR)

14. (a) Examine whether Kolmogorov's S.L.L.N holds for $\{X_n\}$, if $X_n \rightarrow 0$ as X_n's are I.I.D random variables.

(b) Derive the distribution of compound binomial.

15. (a) Define chi-square distribution. Obtain its m.g.f and hence find its mean and variance.

(b) Let $x_1, x_2, \ldots\ldots$ are I.I.D random variables of the continuous type with p.d.f F 'F'. Obtain the p.d.f of $X_{(j)}$ and $X_{(k)}$. also find the marginal p.d.f of $X_{(r)}$.

(OR)

16. (a) Define the maximum and minimum order statistics. Derive the distribution of r^{th} order statistics.

(b) Find the p.d.f of r^{th} order statistics in random sample of size 'n' from exponential (θ) distribution.