35053

M.Sc. DEGREE EXAMINATION, NOVEMBER 2016.

THIRD SEMESTER

Mathematics

Paper III — SEMIGROUPS

Time : Three hours | Maximum : 75 marks

(No additional sheet will be supplied)

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

Each questions carries 5 marks.

Each answer should not exceed 1 page.

1. If G is a group, then show that $\rho \circ \sigma = \sigma \circ \rho$ for any two congruences ρ, σ on G.

2. If S is a relation on a set X, then show that S^{∞} is the smallest transitive relation on X containing S.

3. Show that in S, $(a_i, \lambda)\ \mathcal{L}\ (b, j, l)$ iff $\lambda = l$.

4. Show that se is a 0–minimal left ideal.

5. Define a 0 – simple semigroup. If M is 0 – minimal ideal of S then show that either $M^2\{0\}$ or M is a 0-simple semigroup.

6. Show that a modular lattice is semi modular.

7. Show that the relation $\leq$ defined above is a partial order relation on the inverse semigroup S.

8. Show that any well –ordered chain is anti–uniform.

PART B — ($4 \times 12\frac{1}{2}$ = 50 marks)

Answer ALL questions.

Each question carries $12\frac{1}{2}$ marks.

Each answer should not exceed 5 pages.

UNIT I

9. (a) If R in a binary relation on X, then show that with the usual notation $R^e = [R \cup R^{-1} \cup 1_x]^{\infty}$.

(b) If E is an equivalence relation on S then show that E^{∇} is the largest congruence on S contained in E.

Or

10. (a) If ρ and σ are congruences on a semigroup S such that $\rho \subseteq \sigma$ then show that $\sigma/\rho = \{(x\sigma, y\rho)/\in s/exs/\rho/(x,y) \in \sigma\}$ is a congruence on s/ρ such that $(s/\rho)/(\sigma/\rho)$ is isomorphic to s/σ.

(b) Show that $(\mathcal{B}\ (x), 0)$ is a semi – group.

UNIT II

11. (a) State and prove green's theorem.

(b) Show that a regular semigroup is a group if and only if it has exactly one idempotent.

Or

12. (a) Show that if 'a' is a regular element of a semi group S, then every element of the $\mathcal{D}$– Class D_a is regular.

(b) State and prove Ree's theorem.

UNIT III

13. (a) If S is a union of groups and is simple, then show that it is completely simple.

(b) Show that a band B is a rectangular band if and only if $\mathcal{D} = B \times B$ and is a left zero semigroup if and only if $\mathcal{L} = B \times B$.

Or

14. Show that a band B is normal iff it is a strong semi lattice of rectangular bands.

UNIT IV

15. Show that an inverse semigroup S with semilattice of idempotents E is simple iff $(\forall e, f \in E)\ (\exists g \in E)\ [g \leq f]$ and $e \mathcal{D} g]$

Or

16. (a) Show that a semilattice E has the property that every inverse semigroup having E as a semilattice of idempotents is a Clifford semigroup if and only if E is anitiuniform.

(b) Show that if S is an inverse semigroup, then $\{(x,y) \in s \times x / cs = cy$ for some idempotent C of $S\}$ is the minimum group congruence on S.